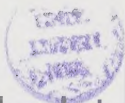


DETECTION AND ESTIMATION IN COMPUTER-BASED ECG ANALYSIS

LEIF SÖRNMO



LUND, SWEDEN
1984



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DETECTION AND ESTIMATION IN COMPUTER-BASED ECG ANALYSIS

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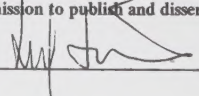
som för avläggande av teknisk doktorsexamen vid tekniska fakulteten
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Lunds Tekniska Högskola, fredagen den 7 december 1984 kl 9.15

av

Leif Sörnmo
tekn.lic.

Organization LUND UNIVERSITY Telecommunication Theory Box 118 S-221 00 LUND / SWEDEN		Document name DOCTORAL DISSERTATION	
		Date of issue December 1984	
		CODEN: LUTEDX/(TETT-1006)/1-166/(1984)	
Author(s) Sörnmo Leif		Sponsoring organization	
Title and subtitle DETECTION AND ESTIMATION IN COMPUTER-BASED ECG ANALYSIS			
Abstract Algorithms for detection and delineation of QRS complexes are found in most systems for computer-based ECG analysis. A model-based approach is applied in this thesis for the purpose of designing such algorithms. QRS detection is treated as a problem of estimating the number of waveforms in an observation interval and their arrival times. The waveforms have random amplitudes and widths and are corrupted by additive Gaussian noise. Two different simplified detectors are described which are derived from the MAP estimation procedure of the unknown quantities. These detectors are made adaptable by utilizing past as well as future waveform properties in determining the criteria for QRS acceptance. A study of detector performance is presented on a material of "difficult" ECGs. Results are obtained for different detection filters as well as for different acceptance criteria. The results indicate that the detectors are able to recognize a wide range of QRS morphologies. QRS delineation is considered as a question of finding changes in the spectral characteristics of the ECG. Based on statistical models for the "low-frequency" and "high-frequency" segments of the ECG the points for the onset and end of the QRS complex are obtained from a maximum-likelihood procedure. The accuracy of the delineation scheme is studied in terms of agreement with manual delineations. The performance is compared to those of envelope-based delineation and template waveform delineation.			
Key words Ambulatory ECG monitoring, Arrival time estimation, Computer-based ECG analysis, Digital signal processing, ECG modelling, Electrocardiography, QRS detection, QRS delineation.			
Classification system and/or index terms (if any)			
Supplementary bibliographical information		Language English	
ISSN and key title 0280-7092		ISBN	
Recipient's notes		Number of pages 166	Price
		Security classification	
Distribution by (name and address)			

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DETECTION AND ESTIMATION IN COMPUTER-BASED ECG ANALYSIS

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DECEMBER 1984

DETECTION AND ESTIMATION IN COMPUTER-BASED EEG ANALYSIS

BY
ANDERS LÖNN



Printed in Sweden
Studentlitteratur
Lund 1984

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ACKNOWLEDGEMENTS

I wish to express my sincere gratitude to my family and to former and present colleagues at

- Department of Telecommunication Theory, University of Lund
- Department of Clinical Physiology, University of Lund
- Department of Medical Informatics, Linköping University
- Department of Clinical Physiology, Central Hospital, Västerås
- Department of Electrical Measurements, University of Lund

for generous support during this work.

GENERAL INTRODUCTION

I. BACKGROUND

Computer techniques have become indispensable in many areas of ECG analysis. One important reason for this development has been the aim to reduce tire-some visual monitoring of ambulatory ECG recordings. Computer techniques provide an efficient means for detection and quantification of arrhythmia episodes that occur during the course of an ECG recording. Data storage on a computer disk allows access to any time segment of an ambulatory recording within a fraction of a second; a facility which is not possible in conventional analog replay systems. Yet another reason for employing computers is to take advantage of digital signal processing techniques for the purpose of reducing noise in recordings performed during exercise. By letting the patient cycle at progressively increasing work loads, information can be gained from the ECG about the presence or absence of myocardial ischemia. At high work loads, however, noise due to muscular activity disturbs the ECG to such an extent that measurements from the original signal are unreliable. To obtain a signal better suited for analysis various means for noise reduction have been suggested, the most widely used of which is time-coherent averaging of similar-shaped heart-beats.

Most systems for computer-based ECG analysis have a set of algorithms in common, irrespective of the area of application (i.e. ambulatory monitoring, on-line monitoring of patients in a coronary care unit or exercise ECG analysis). A basic task in any system is the detection of QRS complexes, because the output of the QRS detector is required for all further analysis (the major deflection of the heart-beat is denoted the QRS complex). In particular, the widespread interest in ambulatory monitoring has emphasized the importance of reliable QRS detection. The demands on QRS detector performance in ambulatory monitoring are high; QRS complexes with a broad spectrum of morphologies should be detected in the presence of different physiological and technical "disturbances". Since a recording typically lasts 24 hours, it is crucial to design a computationally feasible detector in order to arrive at a reasonable over-all processing time of the analysis system. Figure 1 demonstrates the signal processing steps employed in computer-based analysis of ambulatory recordings. The "post-processor" includes all further algorithmic processing of the ECG signal, which usually is synonymous to delineation, characterization and classification of the QRS complex and/or RR-interval

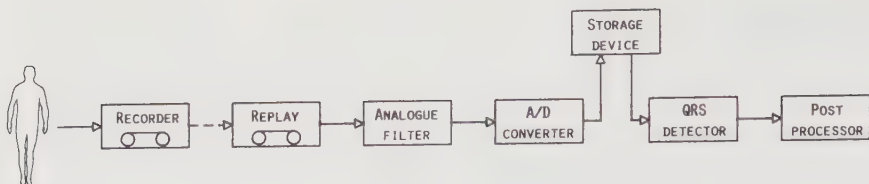


Fig. 1. Block diagram of signal processing steps in off-line analysis of ambulatory ECG recordings.

analysis. (RR denotes the interval between successive beats.) If the post-processor performs multi-waveform classification, the demands on the QRS detector are quite different from those when the post-processor is simply an algorithm for R-R interval analysis. The waveform classifier can typically accept more false detections from the QRS detector than the R-R interval analyser.

A model-based approach to QRS detection and delineation is put forward in this thesis. A stochastic, parametric model is used for each of these purposes. Both models are phenomenological in nature and describe the essential behaviour of a single-lead ECG as obtained from an arbitrary lead configuration. A substantial part of the thesis is concerned with the performance of the resulting algorithms. The performance is studied by means of both real and simulated ECG signals.

The work on QRS detection (Part I and II) was presented in part at Computers in Cardiology [1], [2]. Part I has been published in [3] and Part II has been submitted for publication in IEEE Trans. Biomedical Engineering [4]. The work on QRS delineation (Part III) was presented in part at the 6th Nordic Meeting of Medical and Biological Engineering [5].

II. SIGNALS AND NOISE

The ECG signal reflects the electrical activity of the heart as viewed from electrodes placed on the body surface. The various phases in a heart-beat cycle, i.e. depolarization/repolarization of atria and ventricles, are manifested as a series of deflections which by convention are designated as the P, Q, R, S and T waves, see Fig. 2. The detailed waveform depends among other things on the position of the recording electrodes. The magnitudes and configurations of the waves may, furthermore, serve as an indicator of the heart condition. Another indicator is provided by the pattern of successive R-R intervals.

The intervals are under healthy conditions of almost equal length, although sometimes perturbed by natural breathing. Irregular R-R interval patterns (arrhythmias) may indicate the presence of heart disease. Certain types of arrhythmia are also coupled to changes in beat morphology.

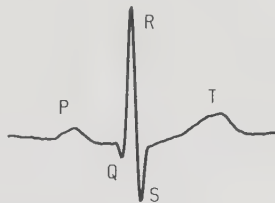


Fig. 2. Definition of P, Q, R, S and T waves.

Signal problems in ECG recordings are, with particular reference to QRS detection, of physiological and technical origin. They can be classified into two main categories, viz. morphology changes (including amplitude changes) and occurrence of disturbances. These categories can be further subclassified:

I. Changes in QRS morphology

- a) of physiological nature
- b) due to technical problems

II. Occurrence of disturbances

- a) myopotentials
- b) transient artifacts (mainly due to electrode problems)
- c) large P- or T-waves

Each main category contains problems of physiological as well as technical nature.



Morphology changes are demonstrated by Figs. 3(a)-(c). In Fig. 3(a) every third QRS complex is quite different in morphology from the others although roughly equal in amplitude. In Fig. 3(b) two morphologies occur; one is remarkably much lower in amplitude than the other and biphasic. These two examples represent premature ventricular complexes (PVCs) and are thus of physiological origin. Fig. 3(c) demonstrates a technically mediated, rather drastic variation in QRS amplitude.

Various common types of disturbances are indicated by Fig. 3(d)-(g). Fig. 3(d) shows a short burst of noise, probably of muscular origin. It is noted that some of the wave shapes in the noise episode are "QRS like", which makes them rather difficult to deal with. Fig. 3(e) shows an example of an even more difficult type of artifact - probably due to electrode problems. The shape and amplitude of the artifacts are fairly typical for QRS complexes and only the time relationships and the complete absence of "T-wave" reveals to the trained eye that they are not QRS complexes. Finally, Figs. 3(f) and (g) show situations where P- and T-waves could be misinterpreted for QRS complexes by a QRS detector. In Fig. 3(f) tall, pointed and uniphasic P waves precede the rather small and biphasic QRS complexes. In Fig. 3(g) the QRS complexes are very low in amplitude relative to the T waves.

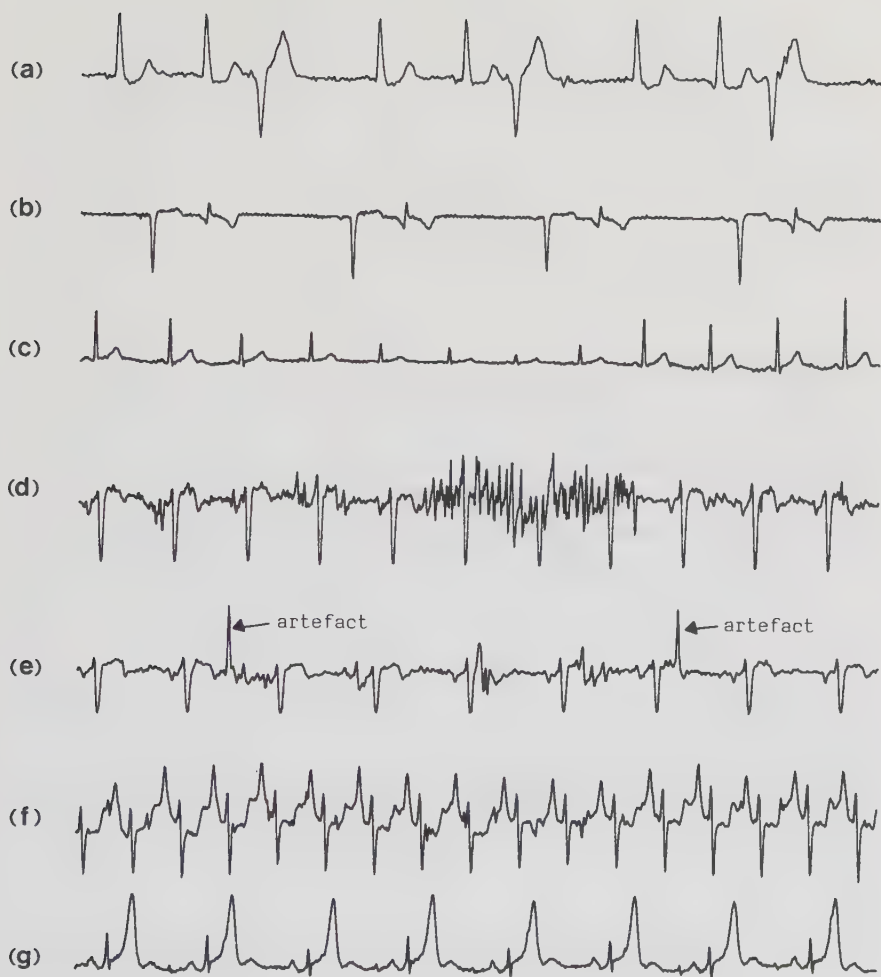


Fig. 3. Various signal problems of physiological and technical origin.
The examples are further discussed in the text.

III. APPROACHES TO QRS DETECTION

Conceptually, most QRS detectors described in the literature can be divided into two entities: the preprocessor and the decision rule (Fig. 4). These two entities are discussed in this section. Since most detectors use a one-channel ECG, the emphasis is on such detectors. The evaluation of performance is briefly considered in Section.III.3. Some QRS detectors incorporate morphological information acquired in the post-processor in order to further improve detector performance [6], [7]. Since such feedback tends to be very system-dependent, it will not be considered. Another subject which is not covered is syntactic QRS detection [8]-[10].

III.1 Digital Preprocessing

The purpose of the preprocessor is to enhance the QRS complexes of the digitized ECG, while suppressing noise and artefacts. Preprocessing can be divided into linear filtering and non-linear transformations. The non-linear transformations consist of e.g. rectification or squaring of the filtered signal. Not all preprocessors employ non-linear transformations; the filtered signal is instead fed directly to the decision rule.

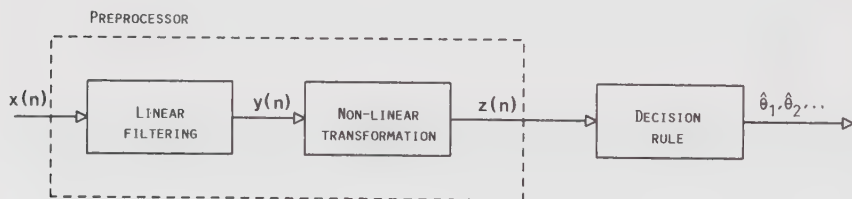


Fig. 4. Block diagram of a QRS detector, composed of a preprocessor and a decision rule. The preprocessor is often subdivided into linear filtering and non-linear transformation. The detector output, $\hat{\theta}_1, \hat{\theta}_2, \dots$, denote estimates of the locations of QRS complexes in time.

a. Linear filtering

The earliest attempts made to produce a signal better conditioned for the decision rule employed the first difference of the ECG [11]-[13]. The same technique has since then been used in several QRS detectors [9], [14]. When analysing resting ECGs, the first difference may perhaps be an acceptable choice. However, it also accentuates high-frequency noise, and is thus not appropriate in situations with moderate or low signal-to-noise ratio (SNR), e.g. in ambulatory monitoring.

Results from the detection of a known signal in Gaussian noise provide a basis for designing a suitable linear filter. It is well-known that the optimum detector includes a matched filter, which is derived from the spectral properties of the QRS complex and the noise. In most situations, this implies the use of a bandpass (BP) filter, since the frequency content of a QRS is essentially in the interval 5-30Hz. In contrast to signals encountered in communication theory, however, the QRS morphology often varies with time and the noise is not stationary. The design of a linear, time-invariant BP-filter must therefore not impose any strict requirements on the transition regions of the frequency response of the filter, but instead try to reconcile the different conflicting demands made upon it. For example, the lower cut-off frequency should be chosen so as to minimize the influence of large-amplitude P and T-waves while still accentuating ectopic beats. Further, the upper cut-off frequency should be chosen so as to suppress motion artefacts but not narrow QRS complexes. Due to the rather loose requirements on the frequency response, simple structured filters can be successfully used. This is of major importance in real-time monitoring and when analysing long-term ECG recordings, since the number of calculations associated with a certain filter is decisive for the total processing time.

The use of a digital BP-filter in the preprocessor is described in many recent papers on detectors. The filter has a centre frequency between 10 and 25Hz, a bandwidth of 5 to 10Hz and is usually implemented with a finite impulse response (FIR) structure. A class of simple digital filters is suggested in Part I. Filters belonging to this class have been used by several authors. The impulse response of each filter is defined by two integer parameters K and L

$$h(k) = Z^{-1} \left\{ \left(1 - z^{-K} \right) \left(1 + z^{-1} \right)^L \right\} \quad (3.1)$$

where $Z^{-1}\{\cdot\}$ is the inverse Z-transform. In the time domain, the first part $(1 - z^{-K})$ forms a difference between the input signal and the delayed input (K samples), and the second part $(1 + z^{-1})^L$ is a lowpass filter with decreased bandwidth for increased L . The filters in this class have linear phase, and they can be implemented solely with additions and subtractions. The above mentioned first difference is given by $(K, L) = (1, 0)$. It has been found that, for the sampling rate $f_s = 100\text{Hz}$, a good performance was obtained by using $(K, L) = (1, 2)$, see Part II. These parameters yield a BP-filter with $f_c = 20\text{Hz}$ and a rather large bandwidth (Fig. 5). Since there is a large spectral variability in QRS complexes a smaller bandwidth will increase the number of beats missed. By using a filter with lower centre frequency, e.g. $(K, L) = (4, 3)$, it was found that the number of false detections due to T-waves was considerably increased. For $f_s = 100\text{Hz}$, the filter $(K, L) = (1, 1)$ has been suggested in [15] and [16]. For a higher sampling rate, i.e. $f_s = 250\text{Hz}$, the filter $(K, L) = (5, 4)$ appears to be an appropriate choice, also being a filter with $f_c = 20\text{Hz}$ (Fig. 6) [17]. By using any of the above mentioned combinations of sampling rate and filter parameters, the mains frequency 50Hz will be cancelled. Another digital BP-filter which is useful for preprocessing of the ECG-signal has been presented in [18]. The filter consists of two cascaded filters: the first filter reduces the baseline wandering and cancels the mains frequency, while the second one accentuates the QRS complexes. The first filter is obtained by combining four averaging filters. For the input ECG signal $x(n)$, the output $y(n)$ is then given by

$$y(n) = \frac{1}{K_1} \sum_{m=n-K_1+1}^n \sum_{k=m-K_1+1}^m x(k) - \frac{1}{L_1} \sum_{m=n-L_1+1}^n \sum_{k=m-L_1+1}^m x(k) \quad (3.2)$$

where K_1, L_1 are two integer parameters defining the length of each averaging filter. The second filter falls within the class defined by eq. (3.1) with filter parameters $(K, L) = (1, 3)$. The over-all frequency response for the cascaded filters with a sampling rate of 250Hz and with $(K_1, L_1) = (5, 200)$, is shown in Fig. 6.

Other "matched" filters for QRS detection have been described in [19] and [20].

Several different choices of BP-filters for analogue QRS detectors have been described [21]-[23]. The filters have centre frequencies in the interval 12 to 17Hz, and bandwidths ranging from 6 to 30Hz.

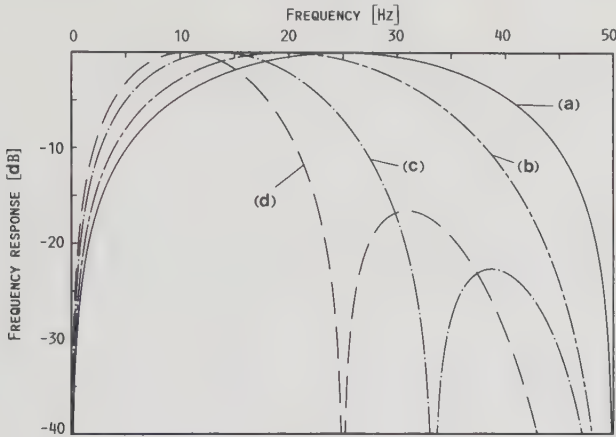


Fig. 5. Frequency response for filters described by parameters K and L , using 100Hz sampling frequency. a) $(K,L) = (1,1)$; b) $(K,L) = (1,2)$; c) $(K,L) = (3,2)$; d) $(K,L) = (4,3)$.

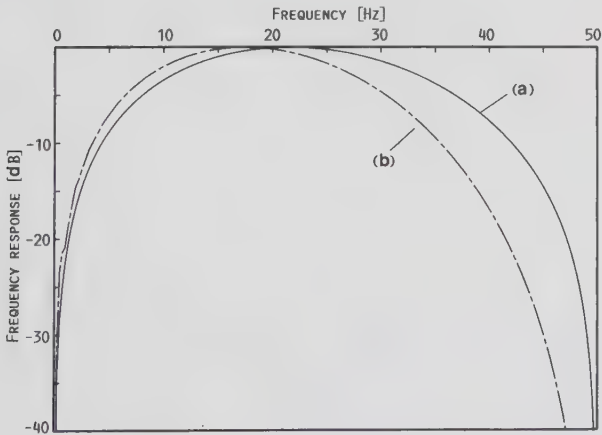


Fig. 6. Frequency response for filters using 250Hz sampling frequency. a) $(K,L) = (5,4)$; b) $(K_1,L_1) = (5,200)$ followed by $(K,L) = (1,3)$.

b. Non-linear transformations

Several heuristic approaches to non-linear transformations of the filtered ECG can be found in the literature, e.g. [24], [25]. A common objective for such approaches is to get a single positive peak for each QRS, which thus enables the use of a peak detector or a one-sided threshold. As is the case with linear filtering, it is also motivated by the desire to obtain a signal in which the QRS complexes are enhanced from the background of P-waves, T-waves, noise and artifacts.

Short-time analysis of a signal is usually accomplished by a memoryless, non-linear transformation on the input signal, followed by linear filtering. In relation to QRS detection, the following transformation has been studied

$$z(n) = \sum_{k=n-N+1}^n y^2(k) h_e(n-k) \quad (3.3)$$

which could be interpreted as a measure of the short-time energy. The signal $y(n)$ may either be the original ECG signal or a filtered version of it, see above. The squared signal is then fed to a filter with a finite impulse response, $h_e(k)$. This filter provides the necessary smoothing of the signal, so that only large-amplitude events of sufficient duration, i.e. the QRS complexes, are preserved in $z(n)$.

Ligtenberg and Kunt [18] calculated the signal $z(n)$ in (3.3), after linear filtering, with a simple averaging filter:

$$h_{e,1}(k) = \begin{cases} 1 & k = 0, \dots, N-1 \\ 0 & \text{otherwise.} \end{cases} \quad (3.4)$$

A smoother behaviour of $z(n)$ could for this filter be obtained by increasing N . Although this reduces the influence of spike artefacts of short duration, it also increases the risk of dropping small PVCs. Another type of filter was proposed by Murthy and Rangaraj [24], who instead filtered the squared, first difference of the ECG with a filter defined by:

$$h_{e,2}(k) = \begin{cases} N-k & k = 0, \dots, N-1 \\ 0 & \text{otherwise.} \end{cases} \quad (3.5)$$

By using the first difference of the ECGs in (3.3) the resulting detector becomes extremely sensitive to high-frequency noise. Since this problem was noted in [24], the filter $h_{e,2}(k)$ was modified to also include averaging.

Even though the ECG signal is not strictly bandlimited, the envelope representation is still advantageous for obtaining a positive-valued signal [26]. The envelope is defined by

$$z(n) = \left(y^2(n) + \hat{y}^2(n) \right)^{1/2} \quad (3.6)$$

where $\hat{y}(n)$ is the discrete Hilbert transform of $y(n)$. An approximation of the ideal Hilbert transform is required before calculation of the envelope. If, furthermore, threshold detection is employed, the ripple in the envelope must be smoothed out. A computationally attractive approximation of (3.6) is given by

$$z(n) \approx |y(n)| + |\hat{y}(n)| \quad (3.7)$$

which, after smoothing, yields a signal similar to the one obtained by (3.6). Brekelmans and de Vaal [27] described a QRS detector, in a multi-channel setting, which preprocesses the BP-filtered signal in essentially the same manner as (3.7).

Although the above mentioned non-linear schemes may be practically suitable, the theoretical basis for these schemes is not evident. A more formal approach for arriving at a non-linear scheme is to formulate a stochastic, parametric model for the ECG, which is the subject of Part I. The specific structure of the non-linear operations, as well as that of the decision rule, is obtained by applying estimation techniques. Briefly, the ECG is modelled as an unknown number, q , of pulse-shaped waveforms $s(k, I_i)$, which have been corrupted by stationary Gaussian noise

$$r(k) = \begin{cases} \sum_{i=1}^q B_i s(k - \theta_i, I_i) + n(k) & 1 \leq q \leq n \\ n(k) & q=0 \end{cases} \quad (3.8)$$

Each waveform occurs at a random time, θ_i , in the observation interval, and has a random amplitude, B_i , and width, T_i . The purpose of the estimator is to find those parameter values of $q, \theta_1, \dots, \theta_q, B_1, \dots, B_q, T_1, \dots, T_q$ which for a given criterion give the best fit for the model to the observed ECG. If a priori knowledge is assumed to exist for the parameters, it is natural to apply the maximum-a-posteriori criterion [28]. The resulting structure of the estimator depends, of course, upon the specific form of the a priori probabilities.

If one assumes a "least informative" situation (i.e. with uniform distributions), the estimation procedure carries out a non-linear transformation on the linearly filtered ECG, which, after some simplifications, is given by

$$z(n) = \max_{\tau_1 \leq T \leq \tau_2} |y(n) - y(n-T)| \quad (3.9)$$

where τ_1 and τ_2 are positive integers. The signal $z(n)$ in (3.9) is thus the rectified, maximal amplitude difference between two samples.

Due to the large computational requirements associated with (3.9), i.e. the maximization is performed at each point in time, n , an approximate scheme may be used in which $y(n)$ and $y(n-T)$ are chosen among the peaks and valleys in the filtered signal, see Part II. Another preprocessing scheme in which $y(n)$ and $y(n-T)$ are chosen among successive peaks and valleys has been investigated in [29] for enhancement of fetal ECG recordings. The use of successive peaks and valleys in a BP-filtered signal can, however, in certain situations, i.e. for split QRS complexes, produce a signal, $z(n)$, in which the complexes are suppressed rather than enhanced, see Part II. A further simplification of (3.9) is obtained by using the absolute value of the difference between two samples separated by a fixed distance $\tau_1 = \tau_2 = \tau$ [30].

Figure 7 demonstrates the output of some non-linear transformations discussed in this section. In each case the input signal (I) is filtered by a filter defined by $(K, L) = (1, 2)$ in (3.1). The sampling frequency is 100Hz. The short-time energy signal (II) is obtained by using the filter in (3.4) with $N=6$. The envelope signal (III) was obtained by (3.6) and a smoothing filter with a triangular impulse response of length 5. An approximate

Hilbert transform was obtained by using a filter of length 15 [26]. The rectified maximal amplitude difference (IV) was calculated with the parameters τ_1 and τ_2 chosen to 2 and 6 respectively. In the approximation of (3.9) by means of a peak-and-valley strategy (V) the same values of τ_1 and τ_2 were used.

It is noted in Fig. 7(a) that the short-time energy (II) produces a signal in which the noise background is better suppressed than in the other three preprocessors. In contrast small-amplitude QRS complexes tend to be too much suppressed, see Fig. 7(b). In Fig. 7(c) it is seen that in the short-time energy signal most T-waves are even larger than the QRS complexes. Note that some T-waves are not represented at all in the peak-and-valley signal (V) due to the width criteria.

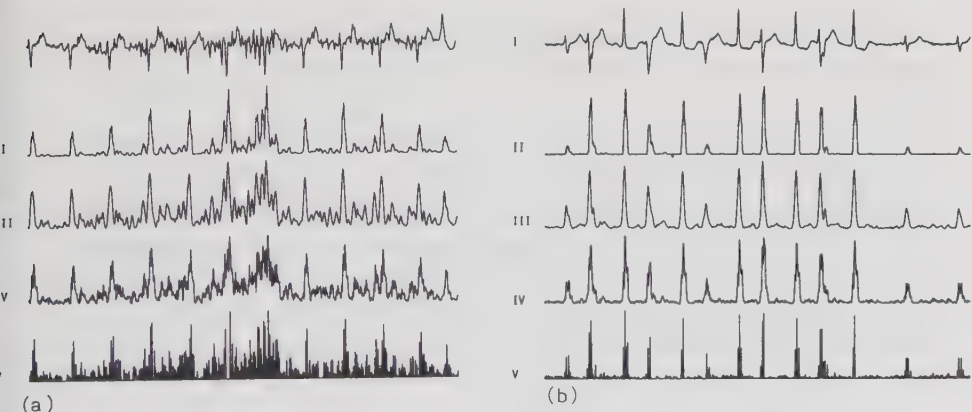
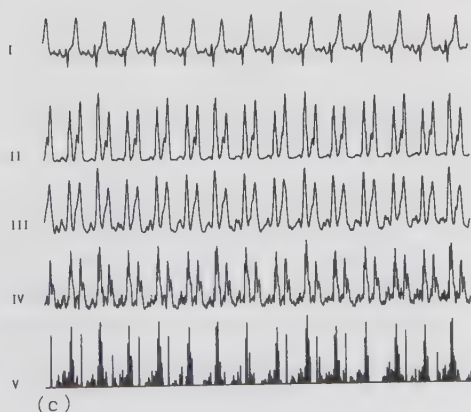


Fig. 7. Three examples of QRS detector input and the output of four different preprocessors.

- I is the input ECG signal
- II is the short-time energy signal
- III is the smoothed envelope signal
- IV is the rectified, maximal amplitude difference between two samples
- V is the signal obtained by a peak-and-valley strategy.

For further clarification, see text.



III.2 Decision Rules

A decision rule is applied to the output of the preprocessor to determine whether or not a QRS complex has occurred. The rule, which often involves an amplitude threshold, may be fixed, or incorporate some kind of adaptivity. Although there are recent proponents for using a fixed threshold [14], adaptive thresholds are now by far the most commonly applied. This is due to the fact that in ambulatory ECG recording, the QRS amplitude and morphology can change drastically during a short time interval; for an example, see Fig. 3. The detection of low-amplitude QRS complexes with a fixed threshold inevitably implies accepting several false detections. Besides comparison of the amplitude in the preprocessed signal to a threshold, the width of the waveform often is part of the basis for decision.

An inherent property of most detectors is their restrictive use of information available from the pattern of preceding RR-intervals. Even though a regular rhythm has prevailed for a long time, the next QRS to be detected is treated as if it could occur almost anywhere in the observation interval. The signal is usually processed in a "causal" fashion, which means that QRS complexes are detected in temporal order, although the whole recording may be available to the detector (i.e. from a computer disc). Several detectors described in the literature make use of a time-dependent threshold in order to reject large T-waves but still allow small PVCs to be detected. The threshold can be represented by

$$\eta(k) = \begin{cases} \alpha_1 & k = \Theta+1, \dots, \Theta+D_1 \\ f(k) & k = \Theta+D_1+1, \dots, \Theta+D_2 \\ \alpha_2 & k = \Theta+D_2, \dots \end{cases} \quad (3.10)$$

where the function $f(k)$ is such that:

$$\alpha_1 \geq f(\Theta+D_1+1) > \dots > f(\Theta+D_2) = \alpha_2 \quad (3.11)$$

and Θ denotes the time instance for the most recently detected QRS complex. The most common choice of (3.10) involves an "eye-closing" period in which no detection can take place before $\Theta+D_2$, i.e. with $\alpha_1 = \infty$, $D_1 = D_2 - 1$. Further, α_2 is adaptively updated e.g. by making it equal to a fraction of the amplitudes of the most recently detected QRS complexes.

The eye-closing period is usually chosen in the interval 120 to 200ms. The obvious risk of missing early PVCs necessitates choosing not too high values for D_1 . A very short interval, and, of course, the exclusion of eye-closing will, however, increase the number of false detections not only due to T-waves, but also due to wide PVCs. There is thus a trade-off in the choice of the eye-closing period which, as pointed out earlier, depends on whether or not waveform classification is performed. Shah et al. [31] have suggested a compromise between the conflicting demands described above. They use a finite α_1 in (3.10), but still with $D_1 = D_2 - 1$. Another choice of (3.10) is suggested in [20] where $\alpha_1 = \infty$, and $f(k)$ is a linearly decreasing function in the interval $[\theta + D_1 + 1, \theta + D_2]$. While the eye-closing period D_1 is preset, D_2 is related to the length of the average RR-interval, i.e. the function $f(k)$ decreases faster when the heart beats faster. When a tentative QRS complex has been detected, additional thresholds are applied in order to ensure that the detected waveform is not a P-wave.

Another type of decision rule can be found in [17], where the rectified, linearly filtered signal is compared to an adaptive threshold. A QRS complex is detected, if a known pattern of threshold crossings occurs within a certain time interval. The threshold is updated as an exponentially weighted average of the maximum filtered QRS amplitude of previously detected complexes. The use of this technique easily prevents false detections due to baseline shifts.

In this thesis, the detector incorporates to a certain degree both past and future signal properties. An observation interval is delimited within which QRS complexes are detected in their order of magnitude in the pre-processed signal, not in temporal order. Due to this "non-causal" property, eye-closing periods of equal length are applied both before and after each detected beat. The detection threshold is adapted with respect to the properties of the complexes which delimit the interval. This approach allows the detector to find the QRS complexes even when a sudden decrease in amplitude occurs.

III.3 Evaluation of Detector Performance

In the preceding sections various approaches to QRS detection have been outlined. Before a detector can be implemented in a clinical setting, the designer must

1. Determine suitable parameter values, and
2. Evaluate the performance for the set of parameter values chosen.

The choice of parameter values has, for the most part, been an integral part of the algorithmic development, rather than the result of a separate optimization. In doing so, the designer runs the risk of choosing values which are well adjusted to the learning material but not necessarily to subsequent materials. There is usually a large number of parameter values that must be fixed within each detector structure. A separate optimization of all parameters, in connection with some performance measure, implies such a time-consuming amount of computations that it is prohibitive at the present stage of computer-technological development. An obvious way to handle this problem is to optimize only those parameters which have the most profound effect on performance. Other values are fixed by *ad-hoc* decisions; certain values may also be determined on physiological grounds. Although a single set of parameter values is to be ultimately used in actual practice, it is of great interest to investigate the performance as a function of a certain parameter. Remarkably little has, however, been published about the influence of different parameters on performance.

Efforts have been most often directed towards evaluating the clinical system as a whole, rather than at performing a separate evaluation of the QRS detector block [32]. Although such an evaluation does not provide any direct information about the performance of the QRS detector, a large number of missed PVCs for example, might indicate a failure in the detector. Since the performance of the QRS detector is a limiting factor for the over-all system performance [33] a separate evaluation of the QRS detector is of major importance. A number of results from evaluation of QRS detectors are found in the literature [6], [25], [14], [34] - [36], [18]. Different workers have collected their own databases due to the lack of a unified one. A comparison of published results has

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no validity, since e.g. signal quality or the number of aberrant beats may vary widely between test sets. Information concerning the tolerance between estimated occurrence time of the QRS complex and the manually annotated occurrence time is, moreover, not often reported.

The performance of two QRS detectors, which are based on maximum-a-posteriori estimation, is investigated in terms of false and true detection rate in Part II. In order to study the behaviour of each detector for different parameter values, the true detector rate is displayed versus the false detection rate in a so-called receiver operating characteristic (ROC). This allows a system designer to choose an "operating point" in the ROC in order to comply with the demands of the post-processor. The performance is studied by varying those parameters which seem to have the most profound effect on performance, in this case the parameters which determine amplitude and width criteria and the filter parameters (K,L) in (3.1). A material of "difficult" ECGs has been collected which contains various "artefacts" of physiological and technical origin (cf. Sec. II). The purpose of collecting such a material is to point out weaknesses in the QRS detectors.

IV. METHODS FOR QRS DELINEATION

The recognition of the onset and end of QRS complexes is necessary in most types of computer-based ECG analysis. The performance of algorithms for such delineations may influence the performance of the analysis system as a whole.

In the analysis of resting ECGs, QRS delineation is one of the cornerstones of the subsequent measurement programs which measure QRS duration, Q wave duration etc. Inaccurate QRS delineation leads to erroneous measurements and may thus lead to erroneous diagnoses.

In exercise ECG analysis interest is focused on the presence or absence of changes in the ST-segment. ST changes are measured at certain predefined distances from the end of the QRS complex; accurate determination of QRS end is thus essential. The reference level for ST measurements is usually estimated from a segment just before QRS onset, the accurate determination of which is also important. ST-segment analysis is becoming of increasing importance in long-term ECG analysis, see e.g. [37]-[39].

QRS delineation is, moreover, a prerequisite in certain schemes for classification of different QRS morphologies, found in e.g. arrhythmia analysis for ambulatory or coronary care monitoring. In these schemes, the features which characterize the QRS complex are computed with respect to the delineated interval. A summary of features which are based on the recognition of QRS onset and end is found in [40]. Delineation schemes which produce inaccurate estimates of QRS onset and end severely limit the reliability of subsequent classification.

The number of leads recorded greatly influences the accuracy of QRS delineation. In a one-channel recording, it is difficult to achieve accurate delineation since the initial depolarization can be very small in the selected lead. The Q wave amplitude may, furthermore, change from beat-to-beat due to body movements, causing the wave to vanish in certain beats. Another problem is the determination of QRS end for certain beat morphologies in which there is no distinct transition between the S-wave and the ST-segment. The delineation task is facilitated when several leads are available. The lead configuration used will naturally affect the delineation; an orthogonal

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lead configuration is typically the most advantageous one. In order to reduce the problems faced in a one-channel setting, the different leads have commonly been combined into a single delineation function. This function is considered to provide a more stable representation of the initial depolarization.

In a majority of systems, the delineation function, $d(n)$, is obtained by first processing each lead, i , in the following manner,

$$d_i(n) = c_1 |h_1(n) * x_i(n)|^p + c_2 |h_2(n) * x_i(n)|^p \quad (4.1)$$

where $x_i(n)$ is the sampled ECG signal and c_1 and c_2 are positive constants. The definitive delineation function for a group of L leads is then obtained by

$$d(n) = \left(\sum_{i=1}^L d_i(n) \right)^{1/p}. \quad (4.2)$$

We will now briefly consider three different choices of filters ($h_1(k)$ and $h_2(k)$) and constants (p , c_1 and c_2) which can be found in the literature.

The so-called spatial velocity, $d_{SV}(n)$, is in three-lead recordings by far the most commonly employed type of delineation function. This function is obtained by using the filter and parameters given in Table I. The spatial velocity arises naturally in the analysis of ECGs which are recorded with an orthogonal lead configuration (vectorcardiograms). An approximation to $d_{SV}(n)$ using p equal to one is found in e.g. [43].

Delineation function	$h_1(k)$	$h_2(k)$	p	c_1	c_2	References
$d_{SV}(n)$	$\partial(k+l) - \partial(k-1),$ $l = 0 \text{ or } 1$	1	2	1	0	e.g. [41] [42]
$d_{WBI}(n)$	$\partial(k+1) - \partial(k-1)$	$h_1(k) * h_1(k)$	1	>0	>0	[43], [44]
$d_E(n)$	1	$\begin{cases} \frac{2}{\pi} \frac{\sin^2(\pi k/2)}{k} & k \neq 0 \\ 0 & k = 0 \end{cases}$	2	1	1	[26]

Table I. Filters and constants used in the delineation function $d_i(n)$ defined by eq. (4.1).

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Another approach has been proposed in [44], in which a function called "the waveform boundary indicator", $d_{\text{WBI}}(n)$, was computed. The function $d_{\text{WBI}}(n)$ combines approximations of the first and second derivative of the ECGs, see Table I. While the constants c_1 and c_2 were treated as design parameters in [44], they were in [45] related to the center frequency of the BP-filter which preceded the computation of $d_{\text{WBI}}(n)$. Estimates of onset and end obtained from $d_{\text{WBI}}(n)$ served in [44] as indicators of appropriate search regions; the actual onset and end were then estimated from the original ECG. The function $d_{\text{WBI}}(n)$ has been employed both for single- and multiple-lead analysis.

Still another approach is to use the envelope of the ECG as a delineation function, cf. p. 11. This approach has been considered for the analysis of single-lead recordings. The envelope, $d_{\text{E}}(n)$, is obtained by the filter and parameters given in Table I. A useful approximation of the envelope was obtained by using a low-order approximation of $h_2(k)$ in combination with $p=1$, [26].

Since $d(n)$ in (4.2) is a positive-valued function the recognition of QRS onset and end can be done by using a simple threshold technique. Another method is that of template matching in which a template is crosscorrelated to $d(n)$, [46], [43], [47], [48]. The templates describe the average behaviour of $d(n)$ at QRS onset and end, respectively, and are usually derived from a learning set in which the QRS complexes have been manually delineated beforehand. Schemes which comprise any of the delineation functions mentioned above are apt to be vulnerable to noisy signals, in particular if threshold techniques are used. It is thus essential to include some kind of smoothing, either on the ECG [44], [48] or on the delineation function [26], in order to produce less noise-sensitive estimates of QRS onset and end. By using linear, time-invariant filtering, bias in terms of widening of the QRS complex can be introduced. Adaptive filtering seems to provide a means for circumventing this problem [48].

The basis for the QRS delineation scheme presented in part III is a statistical model for the ECG, in which the ECG is decomposed into low-frequency segments, either including the P- or the T-wave, and a high-frequency segment, the QRS complex. The signal properties are both modeled by linear systems which are driven by white noise. In order to provide an appropriate description of the low-frequency segment, both dynamical and statistical properties

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of the model are considered to be simple functions of time. It is, furthermore, assumed that the model ECG signal is corrupted by additive noise. The most likely point in time for when a change between models has occurred, i.e. QRS onset or end, is estimated by using a maximum-likelihood (ML) criterion. QRS onset and end are estimated separately.

The accuracy of the delineation scheme is studied in terms of agreement with the manual delineation. Since the definition of QRS onset and end sometimes is unclear, e.g. due to high-frequency noise, a non-ambiguous manual delineation could not always be defined. The uncertainty associated with the delineation is estimated by letting a number of human readers delineate a QRS complex. A performance measure is then introduced in which the automated delineation is related to the dispersion of the manual ones. With respect to this measure, the ML delineation (MLD) is compared to both envelope-based delineation (ED) and template waveform delineation (TWD). The results indicate that MLD yields a better performance for both QRS onset and end and especially for the latter.

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ADAPTIVE QRS DETECTION BASED ON
MAXIMUM-A-POSTERIORI ESTIMATION

PART I

ABSTRACT

A mathematical model for the occurrence of non-overlapping pulsed waveforms corrupted with coloured Gaussian noise is considered for the purpose of QRS detection. The number of waveforms, the arrival times, amplitudes and widths are regarded as random variables. The joint MAP estimation of all the unknown quantities consists of linear filtering followed by an optimization procedure. A class of filters is introduced which are easy to implement. The mismatching obtained by using this class for detection of model QRS complexes is investigated. The optimization procedure is time-consuming and is modified so that a threshold test is obtained. The model formulation with non-overlapping waveforms leads to an "eye-closing" procedure covering a segment before as well as after an accepted event. Adaptivity of the detector is gained by utilizing past as well as future signal properties in determining the criteria for QRS acceptance.

I. INTRODUCTION

QRS detection in ambulatory ECG recordings presents a difficult problem. The signal is sometimes disturbed by nonstationary noise and it is also subject to variations in amplitude due to time-varying impedance at the electrode-skin interfaces. Another problem arises from the fact that the morphology of "normal" QRS complexes may change during the time of recording due to changes in heart position relative to the electrodes. Such changes can be rapid, i.e. occur almost from one QRS complex to the next, due to rapid positional changes of the patient (e.g. a rapid change from recumbent to standing position). Further, QRS complexes of quite different morphology, such as aberrantly conducted beats or beats of ventricular origin, may be intermingled with the "normal" QRS complexes.

During the last decade, the problem of detecting QRS complexes has been treated by several authors. Reliable QRS detection is of fundamental importance, whether the purpose is to locate tentative QRS complexes for further morphological analysis [1], [2] or to obtain a table of R-R intervals [3], [4]. Most recent detectors are intended for software implementation, which allows a more complex structure than realization in hardware. Since the R-wave portion of the complex usually is easy to distinguish due to its relatively sharp peak, several algorithms incorporate simple slope criteria [5], [6]. Commonly, the peaks in the differentiated signal and the duration between these peaks is compared to thresholds, which are successively updated from past signal properties. Simple digital filters have been used in order to avoid power line interference and to reduce the influence of muscle artifacts and baseline wandering [1], [7], [8]. Application of matched filters is discussed in [8], [9] where also the problem of defining an accurate fiducial point is considered.

Little theoretical work has been presented in the field of QRS detection. In the early seventies Haywood et.al. considered a mathematical model for the purpose of detecting ventricular extrasystoles (VES) [10], [11]. In that work the number of waveforms in the observation interval was known and the amplitude and arrival time were considered to be random quantities. The waveforms were additively corrupted by stationary, white Gaussian noise. Based on the model, an optimal detection scheme was derived and a number of approximations was presented. Considerations in the application of their detector to real ECG signals seem, so far, not to have been pub-

lished. We have in this paper started with a model formulation which has similarities to the problem of resolving different targets, as treated in radar theory [12] - [14]. In that case the object is to detect or estimate an unknown number of received waveforms and their arrival times.

In Section II, a mathematical model for QRS detection is presented. The model includes the most common properties of an ECG, e.g. possible changes in amplitude and width. A detailed modelling of the ECG is not motivated, since the detector must perform well on a large variety of signal types, cf. p. 5. Based on this model, the MAP criterion is considered for estimating the number of waveforms and their arrival times. In order to arrive at a reasonable amount of computations, an approximate estimation procedure is introduced. In Section IV, different aspects of the model, such as the effect of using filters mismatched to the input signal, are discussed in relation to ECGs. Further the observation interval is delimited by means of two "significant" waveforms. By letting the properties of these waveforms determine the acceptance criteria within the interval, the detector adapts quickly to changes in QRS morphology. Also, such a delimitation is a means for utilizing both past and future signal properties. Since the detector is based on MAP estimation, the concepts 'detector' and 'estimator' are used interchangeably.

II. JOINT ESTIMATION OF THE NUMBER OF WAVEFORMS AND THEIR ARRIVAL TIMES

II.1 Signal model

In this section we consider the following discrete time model formulation; a finite sequence of variables $r(k)$, $k = 1, 2, \dots, N$, also denoted by the N dimensional vector $\underline{r}$, is observed, containing an unknown number, q , of pulsed waveforms, corrupted by additive, white Gaussian noise $w(k)$ with the spectral density $N_0/2$,

$$r(k) = \begin{cases} \sum_{i=1}^q B_i s(k - \theta_i, I_i) + w(k) & 1 \leq q \leq n \\ w(k) & q = 0. \end{cases} \quad (2.1)$$

Each waveform belongs to a known class of signals $(\sim s(k, \cdot))$, but with unknown arrival time θ_i , amplitude B_i and width I_i . The waveform $s(k, I_i)$ has a duration less than D and is composed of two equal waveforms $v(k)$,

$$s(k, I_i) = \begin{cases} v(k) - v(k + I_i) & 0 \leq k \leq D - 1 \\ 0 & \text{otherwise.} \end{cases} \quad (2.2)$$

The parameters q , $\underline{\theta}$ and $\underline{I}$ are discrete-valued and $\underline{B}$ continuous-valued random variables. The joint *a priori* probability density is denoted by $p(q, \underline{\theta}, \underline{B}, \underline{I})$. Since q is constrained to the interval $0 \leq q \leq n$, it is sufficient with n components in each of the vectors $\underline{\theta}$, $\underline{B}$ and $\underline{I}$.

The arrival times $\theta_1, \theta_2, \dots, \theta_q$ are separated by at least the distance D , i.e.

$$|\theta_i - \theta_j| \geq D \quad \text{for } i \neq j. \quad (2.3a)$$

Thus the pulsed waveforms are non-overlapping. To guarantee that all waveforms are completely contained in the observation interval $1 \leq k \leq N$,

$$1 \leq \theta_i \leq N - (D - 1) \quad i = 1, \dots, n \quad (2.3b)$$

we require that q fulfils

$$q \leq n \leq \left\lfloor \frac{N-D}{2D-1} + 1 \right\rfloor, \quad (2.4)$$

where $\lfloor \cdot \rfloor$ denotes the integer part. With the constraint (2.4) it is possible to assume that q and $\underline{\Theta}$ are independent. In our model all the random variables are independent, apart from an interdependence between the components of $\underline{\Theta}$. Then the joint probability density can be written

$$p(q, \underline{\Theta}, \underline{B}, \underline{I}) = \Pr(q) \Pr(\underline{\Theta}) \prod_{i=1}^n p(B_i) \Pr(I_i). \quad (2.5)$$

The probability for q waveforms is

$$\Pr(q) = \begin{cases} p_q & q = 0, 1, 2, \dots, n \\ 0 & \text{otherwise.} \end{cases} \quad (2.6)$$

The probability density for the amplitude B_i is double-sided uniform, see Fig. 1

$$p(B_i) = \frac{1}{2(\beta_2 - \beta_1)} \chi_B(|B_i|) \quad i = 1, 2, \dots, n \quad (2.7a)$$

where

$$\chi_B(|B_i|) = \begin{cases} 1 & 0 < \beta_1 \leq |B_i| \leq \beta_2 \\ 0 & \text{otherwise.} \end{cases} \quad (2.7b)$$

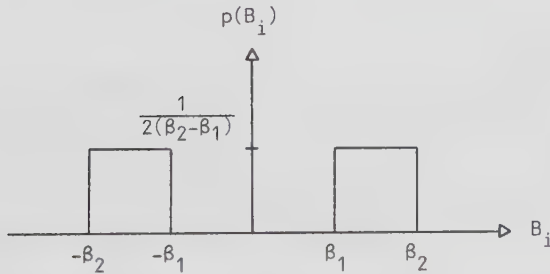


Fig. 1. The probability density for B_i .

The a priori probability for T_i is uniform,

$$\Pr(T_i) = \frac{1}{\tau_2 - \tau_1} X_T(T_i) \quad i = 1, 2, \dots, n \quad (2.8a)$$

where

$$X_T(T_i) = \begin{cases} 1 & \tau_1 \leq T_i \leq \tau_2 \\ 0 & \text{otherwise.} \end{cases} \quad (2.8b)$$

The parameters τ_1 and τ_2 are positive integers. The joint probability $\Pr(\underline{\theta})$ is uniform for all $\underline{\theta}$ which fulfil (2.3),

$$\Pr(\underline{\theta}) = C_n X_{\theta}(\underline{\theta}) \quad (2.9a)$$

where

$$X_{\theta}(\underline{\theta}) = \begin{cases} 1 & \left\{ \begin{array}{l} |\theta_i - \theta_j| \geq D \quad i \neq j \quad i, j = 1, 2, \dots, n \\ \text{and} \\ 1 \leq \theta_i \leq N - (D - 1) \quad i = 1, 2, \dots, n \end{array} \right. \\ 0 & \text{otherwise.} \end{cases} \quad (2.9b)$$

The constant C_n is independent of $\underline{\theta}$ but depends on the dimension n and is given by the condition that $\Pr(\underline{\theta})$ sums up to unity.

II.2 MAP estimation

Based on the a priori knowledge $p(q, \underline{\theta}, \underline{B}, \underline{I})$ and the observed signal vector $\underline{r}$, we want to estimate the number of pulses q in the observation interval and their arrival times $\theta_1, \theta_2, \dots, \theta_q$. To do this, we use the joint MAP estimates of all the unknown parameters q , $\underline{\theta}$, $\underline{B}$ and $\underline{I}$. These joint MAP estimates are given by any $(\hat{q}, \hat{\underline{\theta}}, \hat{\underline{B}}, \hat{\underline{I}})$ that maximizes the log likelihood function [15],

$$\begin{aligned} V(\underline{r}, q, \underline{\theta}, \underline{B}, \underline{I}) = & \frac{2}{N} \sum_{k=1}^N r(k) \sum_{i=1}^q B_i s(k - \theta_i, I_i) - \frac{1}{N} \sum_{k=1}^N \left(\sum_{i=1}^q B_i s(k - \theta_i, I_i) \right)^2 \\ & + \ln p(q, \underline{\theta}, \underline{B}, \underline{I}), \end{aligned} \quad (2.10)$$

where the sums over i equal zero for $q=0$. The first term in (2.10) can be obtained as a weighted sum of outputs from a linear, time-invariant filter with the impulse response

$$h_0(k) = \frac{v(-k)}{2E_v} \quad (2.11)$$

where E_v is the energy of $v(k)$

$$E_v = \sum_{k=0}^{D-1} v^2(k). \quad (2.12)$$

The output of the filter at the discrete time θ_i is given by the convolution sum

$$y(\theta_i) = \sum_{k=1}^N r(k) h_0(\theta_i - k). \quad (2.13)$$

In order to arrive at a convenient form for $V(\underline{r}, q, \underline{\theta}, \underline{B}, \underline{I})$ we introduce the following notations. The signal-to-noise ratio (SNR) $d^2(I_i)$ for $B_i=1$ is given by

$$d^2(I_i) = \frac{2}{N_0} \sum_{k=1}^N s^2(k, I_i) = d_0^2 (1 - \rho(I_i)) \quad (2.14)$$

where

$$d_0^2 = \frac{4E_v}{N_0} \quad (2.15)$$

and $\rho(I_i)$ is the normalized correlation sum for $v(k)$,

$$\rho(I_i) = \frac{1}{E_v} \sum_{k=0}^{D-1} v(k) v(k+I_i). \quad (2.16)$$

By means of the new notations and the a priori probabilities, the log-likelihood function can be rewritten

$$\begin{aligned} V(\underline{r}, q, \underline{\theta}, \underline{B}, \underline{I}) = & \sum_{i=1}^q d^2(I_i) \left[B_i \frac{y(\theta_i) - y(\theta_i - I_i)}{1 - \rho(I_i)} - \right. \\ & \left. - \frac{B_i^2}{2} + \frac{1}{d^2(I_i)} \ln X_B(|B_i|) \right] + \ln p_q + \ln X_{\underline{\theta}}(\underline{\theta}) + \\ & + \sum_{i=1}^q \ln X_I(I_i) + \sum_{i=q+1}^n \ln X_B(|B_i|) X_I(I_i) \end{aligned} \quad (2.17)$$

where we have omitted an additive constant.

We now first maximize (2.17) with respect to $\underline{B}$, $\underline{I}$ and $\theta_{q+1}, \dots, \theta_n$. The last term is either equal to zero or negative infinity and thus, to maximize $V(\underline{r}, q, \underline{\theta}, \underline{B}, \underline{I})$ it should be zero. Since we could always choose $\theta_{q+1}, \theta_{q+2}, \dots, \theta_n$ such that $\ln X_{\underline{\theta}}(\underline{\theta})$ also equals zero, $\ln X_{\underline{\theta}}(\underline{\theta})$ is replaced by $\ln X_{\underline{\theta}}(\theta_1, \theta_2, \dots, \theta_q)$, which is given by (2.9b) with n replaced by q . All terms in (2.17) which depend on B_i , with index $i \leq q$, have been collected in square brackets. In Appendix A this expression is maximized with respect to B_i resulting in a function $\hat{B}_i(\theta_i, I_i)$ which is then substituted for B_i in (2.17). The joint MAP estimates $\hat{q}$, $\hat{\underline{\theta}}$, $\hat{\underline{I}}$ and $\hat{\underline{B}}$ ($\hat{B}_i = \hat{B}_i(\hat{\theta}_i, \hat{I}_i)$) are obtained by maximizing the log-likelihood function

$$V(\underline{r}, q, \underline{\theta}, \underline{I}) = \sum_{i=1}^q d^2(I_i) \beta_1 \left[F \left(\frac{y(\theta_i) - y(\theta_i - I_i)}{1 - \rho(I_i)} \right) - \frac{\beta_1}{2} \right] + \\ + \ln p_q + \ln X_{\underline{\theta}}(\theta_1, \theta_2, \dots, \theta_q) + \sum_{i=1}^q \ln X_I(I_i), \quad (2.18a)$$

where

$$F(x) = \begin{cases} |x| & |x| < \beta_1 \\ \frac{|x|^2 + \beta_1^2}{2\beta_1} & \beta_1 \leq |x| \leq \beta_2 \\ \frac{2|x|\beta_2 + \beta_1^2 - \beta_2^2}{2\beta_1} & |x| > \beta_2. \end{cases} \quad (2.18b)$$

Further, to eliminate I_i in an analogous manner, we introduce the function $M(\theta_i)$, defined by

$$M(\theta_i) = \max_{I_i} \left[\left(1 - \rho(I_i) \right) F \left(\frac{y(\theta_i) - y(\theta_i - I_i)}{1 - \rho(I_i)} \right) + \right. \\ \left. + \rho(I_i) \frac{\beta_1}{2} + \frac{1}{d_0 \beta_1} \ln X_I(I_i) \right]. \quad (2.19)$$

In this case we cannot find an explicit expression for the I_i which maximizes (2.19). The calculation of $M(\theta_i)$ is illustrated in Fig. 2 for $I_i \in [2, 4]$.

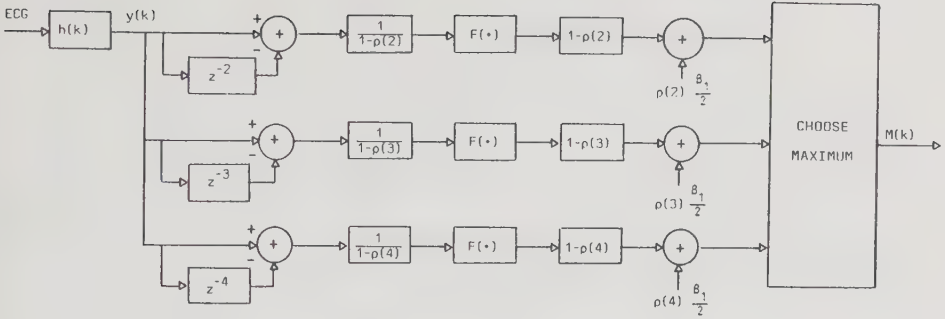


Fig. 2. Preprocessing of the ECG to calculate the function defined by (2.19). In the figure the width I_1 is constrained to the values 2, 3 or 4.

Using (2.19) we can maximize

$$V(\underline{r}, q, \underline{\theta}) = d_0^2 \beta_1 \sum_{i=1}^q \left[M(\theta_i) - \frac{\beta_1}{2} \right] + \ln p_q + \ln X_{\theta}(\theta_1, \theta_2, \dots, \theta_q) \quad (2.20)$$

to obtain the joint MAP estimates. When maximizing (2.20), which contains the quantities of primary interest, q and $\underline{\theta}$, one could first look at a fixed q and determine those $\underline{\theta}$ that yield the maximum. Then, the estimated number of waveforms $\hat{q}$ is equal to that q which gives the over-all maximum.

II.3 Approximate MAP estimation

The calculation of the estimates $(\hat{q}, \hat{\underline{\theta}})$ described in the previous section is time-consuming, particularly for large n . Thus we do not intend to find the optimum, but will be satisfied in finding the estimations $\tilde{q}, \tilde{\theta}_1, \dots, \tilde{\theta}_{\tilde{q}}$ defined by the maximum $V_{\tilde{q}} = \max_{0 \leq q \leq n} V_q$, where V_q is given by the recursion

Adaptive QRS detection

$$\begin{aligned}
 V_0 &= 0 \\
 V_1 &= V(\underline{r}, 1, \tilde{\theta}_1) = \max_{\theta_1} V(\underline{r}, 1, \theta_1) \\
 V_2 &= V(\underline{r}, 2, \tilde{\theta}_1, \tilde{\theta}_2) = \max_{\theta_2} V(\underline{r}, 2, \tilde{\theta}_1, \theta_2) \\
 &\vdots \\
 &\vdots \\
 V_q &= V(\underline{r}, q, \tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_i) = \max_{\theta_i} V(\underline{r}, q, \tilde{\theta}_1, \tilde{\theta}_2, \dots, \theta_i) \\
 &\vdots \\
 &\vdots \\
 V_n &= V(\underline{r}, n, \tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_n) = \max_{\theta_n} V(\underline{r}, n, \tilde{\theta}_1, \tilde{\theta}_2, \dots, \theta_n). \quad (2.21)
 \end{aligned}$$

Thus in each step we only have to determine one parameter, $\tilde{\theta}_i$, which equally is obtained by maximizing the difference

$$\begin{aligned}
 \Delta V(\underline{r}, q, \theta_i) &= V(\underline{r}, q, \tilde{\theta}_1, \tilde{\theta}_2, \dots, \theta_i) - \\
 &- V(\underline{r}, q-1, \tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_{i-1}) \quad 1 \leq i \leq n \quad (2.22)
 \end{aligned}$$

The maximum is denoted by Δ_i ,

$$\Delta_i = V_i - V_{i-1} = \max_{\theta_i} \Delta V(\underline{r}, q, \theta_i) \quad 1 \leq i \leq n \quad (2.23)$$

In terms of Δ_i , V_q can be written

$$V_q = \sum_{i=1}^q \Delta_i \quad 0 \leq q \leq n \quad (2.24)$$

with the sum defined as zero for $q=0$.

By using (2.20) in (2.23) we get

$$\Delta_i = d_0^2 \beta_1 \max_{\theta_i} [M(\theta_i) - \alpha_i(\theta_i)] \quad (2.25)$$

where

$$\alpha_i(\theta_i) = \frac{\beta_1}{2} + \frac{1}{d_0^2 \beta_1} \ln \frac{p_{i-1}}{p_i} - \ln X(\tilde{\theta}_1, \tilde{\theta}_2, \dots, \theta_i). \quad (2.26)$$

Eqn. (2.25) shows that any $M(\theta_i)$ which is larger than $\alpha_i(\theta_i)$ gives a positive contribution to V_q . With equal a priori probabilities for q ($p_i = 1/(n+1)$, $i = 0, 1, \dots, n$), $\alpha_i(\theta_i)$ is independent of q and the procedure is analogous to a threshold test. When $(p_i - p_{i-1})$ is a non-decreasing function of i , $\alpha_i(\theta_i)$ is a non-decreasing function of i . Then we can finish the recursion procedure as soon as we find a negative Δ_i , since the following ones are also negative. Some examples which illustrate the relation between the optimal and the approximate MAP estimation are found in Appendix B.

III. MODIFICATIONS OF THE MODEL

From a mathematical model a method has been developed for estimating the number of pulses in an observation interval and their arrival times. Only the white noise case was studied. We will here generalize the model to a continuous-time situation for bandlimited waveforms corrupted with bandlimited coloured noise. A family of simple power spectra for the noise is used.

In general, estimation in coloured noise is more complex than estimation in white noise. However, for an infinite observation interval and stationary noise, the increased complexity is essentially restricted to exchanging the matched filter. In the bandlimited case a discrete-time estimator, see Fig. 3, will give the same result as a continuous-time one, if the observed signal $r^c(t)$ (c denotes continuous-time) is sampled with at least the Nyquist rate. We now allow continuous-valued arrival times θ_i for the waveforms in the observed signal (2.1). By constraining the estimator to produce only discrete-valued $\hat{\theta}_i$, a degradation in performance is obtained.

The impulse response of the matched filter will depend not only on the configuration of the input signal but also on the power spectrum of the coloured noise. Alternatively, if we specify a certain filter and a certain noise spectrum the filter will be matched to a certain input signal. We will investigate a class of filters with finite impulse response, described by two integer parameters.

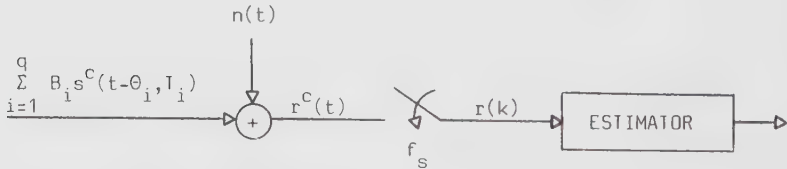


Fig. 3. Extension to coloured noise $n(t)$ and sampling of the observed signal $r^c(t)$ with the sampling rate f_s . ($r(k) = r^c(k/f_s)$ where c denotes continuous-time.)

III.1 Estimator for sampled signals corrupted with coloured noise

We assume that the noise is Gaussian with the power spectrum

$$R_n^C(f) \sim \begin{cases} \left| \frac{1 - be^{-j\frac{2\pi f}{2W}}}{1 - ae^{-j\frac{2\pi f}{2W}}} \right|^2 & |f| \leq W \quad 0 \leq a, b \leq 1 \\ 0 & |f| > W \end{cases} \quad (3.1)$$

where the bandwidth W is so large that the signal energy for $|f| \geq W$ is zero. The family of noise spectra (3.1) enables us to roughly model disturbances of lowpass character. Three spectra in this family are shown in Fig. 4 for various choices of the parameters a and b . Note that $a=b$ gives the white noise treated in Section II.

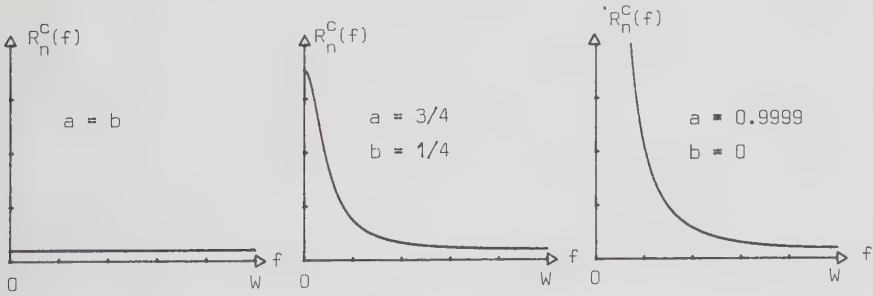


Fig. 4. Noise spectra for three choices of a and b .

Despite the infinite observation interval (i.e. the observed signal $r^C(t)$ is given for $-\infty \leq t \leq \infty$) we restrict $\underline{0}$ to be such that the waveforms are completely contained in a finite interval, just as in Section II. Since $r^C(t)$ is bandlimited and given in an infinite interval, the sampled signal $r(k) = r^C(k/f_s)$ will contain the same amount of information (as long as the sampling rate $f_s > 2W$). Then, in the coloured noise case the log-likelihood function is given by [15]

$$\begin{aligned}
 V(\underline{r}, \underline{q}, \underline{\theta}, \underline{B}, \underline{I}) = & \sum_{k=-\infty}^{\infty} r(k) \sum_{i=1}^q B_i g_0(\theta_i - k, I_i) - \\
 & - \frac{1}{2} \sum_{k=-\infty}^{\infty} \sum_{i=1}^q B_i s(k - \theta_i, I_i) \sum_{j=1}^q B_j g_0(\theta_j - k, I_j) + \\
 & + \ln p(\underline{q}, \underline{\theta}, \underline{B}, \underline{I})
 \end{aligned} \quad (3.2)$$

Note that (3.2) differs from (2.10) not only in that the matched filter $s(k, I_i)$ is replaced by $g_0(k, I_i)$, but also that θ_i is now continuous-valued. The matched filter $g_0(k, I_i)$ is given by the inverse Fourier transform

$$g_0(k, I_i) = F^{-1} \left\{ \frac{S^*(v, I_i)}{R_n(v)} \right\} = \int_{-1/2}^{1/2} \frac{S^*(v, I_i)}{R_n(v)} e^{j2\pi vk} dv \quad (3.3)$$

where $S(v, I_i)$ is the Fourier transform of $s(k, I_i)$,

$$S(v, I_i) = F\{s(k, I_i)\} = \sum_{k=-\infty}^{\infty} s(k, I_i) e^{-j2\pi vk} \quad (3.4)$$

and $R_n(v)$ is the power spectrum for the sampled noise. Since $f_s > 2W$

$$R_n(v) = f_s R_n^C(f_s v) \quad (3.5)$$

with $R_n^C(f)$ given by (3.1). For the purpose of utilizing the results in Section II we redefine some of the quantities defined in that section.

Due to the coloured noise situation, (2.11) is changed to

$$h_0(k) = F^{-1} \left\{ \frac{V^*(v)}{d_0^2 R_n(v)} \right\} \quad (3.6)$$

where d_0^2 is such that

$$d_0^2 = 2 \int_{-1/2}^{1/2} \frac{|V(v)|^2}{R_n(v)} dv. \quad (3.7)$$

Further $\rho(T_i)$ in (2.16) is replaced by

$$\rho(T_i) = 2 \sum_k v(k) h_0(T_i - k). \quad (3.8)$$

We neglect the energy outside the duration D of the waveform $s(k, T_i)$ and the matched filter $g_0(k, T_i)$. Then, with the quantities (3.3)-(3.8) inserted into (3.2) the estimator is almost identical to the one in Section II. The difference is that θ_i in (3.2) is continuous-valued, so (3.6) should be evaluated for continuous k . The estimator could be implemented by using a number, P , of filters in parallel, $h_0(k)$, $h_0(k+1/P), \dots, h_0(k+(P-1)/P)$. For a sufficiently large P we have then implemented an estimator for continuous-valued θ_i . In the following we use $P=1$, i.e. the estimator produces only discrete-valued $\hat{\theta}_i$. This choice leads to two types of errors:

1. An extra error in the estimate of the arrival time is introduced.
2. The maximal possible signal magnitude after the "matched" filter is reduced (loss in SNR).

The first error is at most equal to $1/(2f_s)$. The second error is investigated in Section IV.

III.2 A class of filters

We will now treat the case, when the filter $h_0(k)$ and the power spectrum of the coloured noise in (3.5) are given. The matched filter $g_0(k, T_i)$ in (3.2) is then determined by (3.3), (3.6) and (2.2). Thus

$$g_0(k, T_i) = d_0^2 \{h_0(k) - h_0(k - T_i)\}. \quad (3.9)$$

Utilizing this filter and the power spectrum (3.5) in (3.3) we obtain a unique waveform $s(k, T_i)$ to which $g_0(k, T_i)$ is matched. Since the continuous-time signal is bandlimited, it is uniquely determined as

$$s^c(t, T_i) = \int_{-1/2}^{1/2} R_n(v) G_0^*(v, T_i) e^{j2\pi v f_s t} dv. \quad (3.10)$$

To evaluate (3.9) we must specify the filter $h_0(k)$. We will here consider a class of filters for $h_0(k)$ defined by two integer parameters

K (=1,2,3 or 4) and L (=0,1,2 or 3)

$$h_{KL}(k) = Z^{-1}\{(1-z^{-K})(1+z^{-1})^L z^D\}, \quad (3.11)$$

where $Z^{-1}\{ \}$ is the inverse Z-transform. The frequency function for different combinations of (K,L) are shown on p. 9. The factor z^D is included in the filter to fit the structure in Section II. When implementing the filter, z^D could be left out with the consequence that the output will be delayed by D samples. Using $h_{KL}(k)$ for $h_0(k)$ in (3.9) we obtain the filter

$$g_{KL}(k, I_i) = d_0^2 \{h_{KL}(k) - h_{KL}(k-I_i)\}. \quad (3.12)$$

Since $h_{KL}(k)$ is antisymmetric, $g_{KL}(k, I_i)$ is symmetric. The signals $s^c(t, I_i)$ of Fig. 5 have been obtained by replacing $g_0(k, I_i)$ in (3.10) with $g_{KL}(k, I_i)$. Once K, L and the noise spectrum (a and b in (3.1)) have been chosen, the filter will be matched (apart from the first error) to the sampled versions of waveforms given by varying the parameter I_i (one time axis in the figure). In spite of the simple parametric structure of the filter $h_{KL}(k)$, we can model a fairly wide class of signals. One constraint is that only symmetric signals can be modelled. In the next section, we will further study the filters in the given class, when used on symmetric and antisymmetric sampled signals.

The correlation $\rho(I_i)$ is shown in Fig. 6 for some choices of the filter parameters K and L. For $K \leq 2$ and $I_i \geq 2$, it is seen that $\rho(I_i)$ is close to zero in the coloured noise cases (Fig. 6(b)-6(c)).

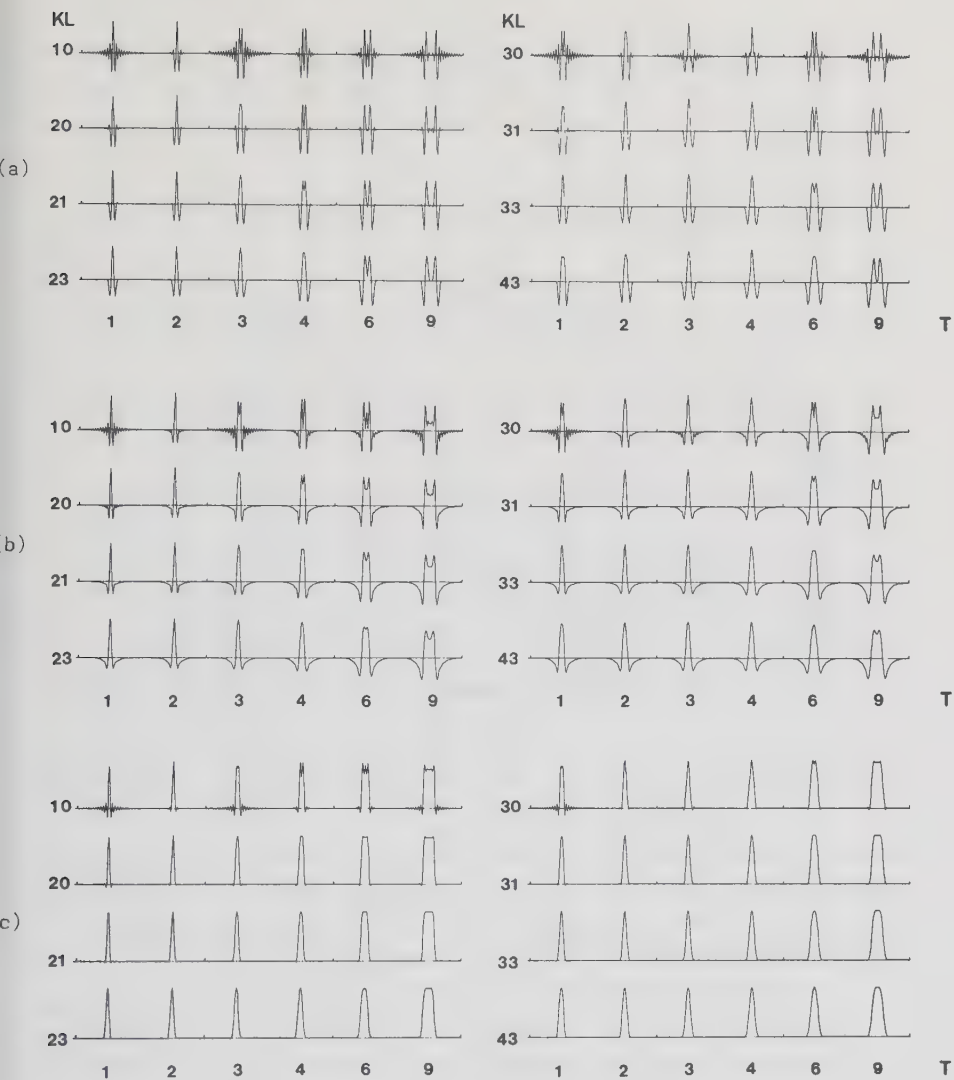


Fig. 5. Waveforms to which the filter $g_{KL}(k, T)$ is matched for 3 different noise spectra.

(a) $a=b$; (b) $a = 3/4$, $b = 1/4$; (c) $a = 0.9999$, $b = 0$.

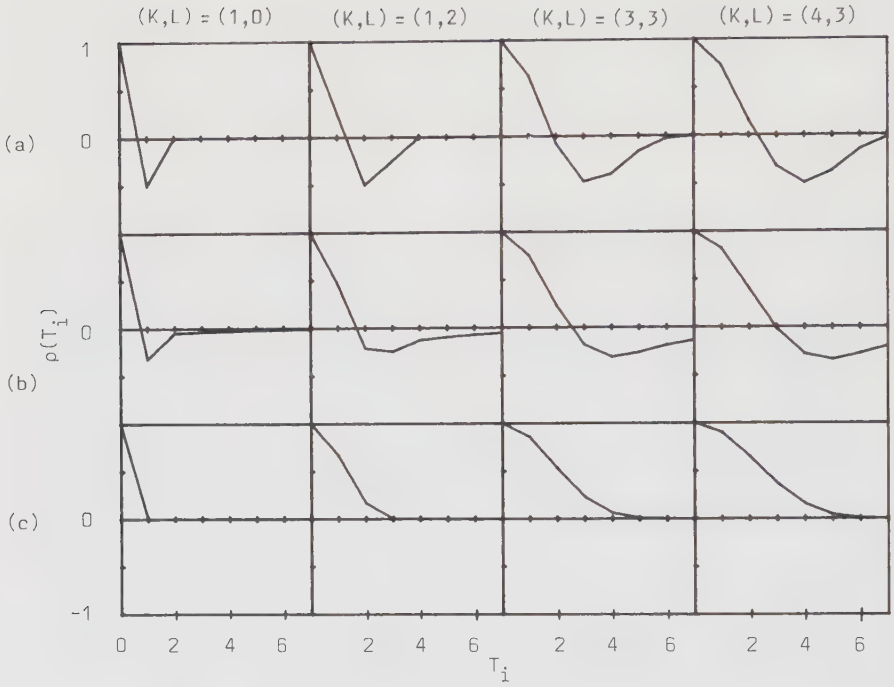


Fig. 6. The correlation function $\rho(T_i)$ for three noise spectra.

(a) $a = b$; (b) $a = 3/4$, $b = 1/4$; (c) $a = 0.9999$, $b = 0$.

IV. APPLICATION OF THE ESTIMATOR TO SAMPLED ECGs

In Section II and III we have developed an estimator, which works on a sampled signal. No reference was made to ECGs. Here, we will apply our estimator to ECGs sampled at $f_s = 100$ Hz. It is appropriate at this point to relate certain properties of the model that we used for determining the estimator to the corresponding ECG properties, see Table 1.

	ECG	Model
1.	not strictly bandlimited	strictly bandlimited
2.	non-stationary, complex noise situation	bandlimited, coloured stationary Gaussian noise
3.	P waves, T waves and artifacts occur	P waves, T waves and artifacts not directly modelled
4.	rhythm most often regular	weak assumptions as regards rhythm
5.	wide class of waveforms	assumes waveforms of type depicted in Fig. 4.
6.	observed waveforms most probably have the same polarity	assumes positive and negative polarity with equal probability

Table 1. Properties of ECGs versus model.

The discrepancy in property 1 is taken care of by prefiltering and sampling. The prefiltering implies that we lose some of the high-frequency characteristics of the signal. Since we can model low-frequency noise, cf. (3.1), important aspects of properties 2 and 3 can be taken care of, namely baseline wandering and T waves. These are in general more low-frequency than QRS complexes. High-frequency noise due to e.g. muscular activity will be attenuated by the prefilter. Our model does not include any high-frequency noise above the white-noise level. In the subsections below we consider properties 4 to 6.

IV.1 Assumptions on rhythm

Most often QRS complexes are fairly regularly spaced along the time axis. Assuming a regular rhythm in the model would probably improve the performance in normal ECGs with sporadic artefacts. However, in our opinion one should not impose a rigid structure on the timing of events since one of the most important qualities of a good QRS detector is to reliably detect *changes* in rhythm. These are often clinically important.

Our assumption of a uniform $\Pr(\theta)$ leads to an estimator which favours no particular locations of QRS complexes in the observation interval. However, we have postulated that they cannot overlap. Unequal a priori probabilities, $\Pr(q)$, for the number of complexes in the observation interval may be used. This allows the estimator to favour a certain heart rate.

IV.2 Mismatched filters

In Section III we suggested a class of filters characterized by two integer parameters. Signals to which these filters could be matched were shown in Fig. 5. It is seen in this figure that in many cases essential properties of QRS complexes are modelled. It is well-known from signal theory [15] that matched filters are often rather insensitive to moderate variations in signal and noise properties. In this subsection we want to study the reduction in performance that results from using filters in the given class ($h_{KL}(k)$) to input signals that are not matched to the filter. We will investigate some of the combinations of K and L used in Fig. 5. As input signals, we have chosen two model QRS complexes with variable widths, $s_I^C(t)$ and $s_{II}^C(t)$ given by

$$s_I^C(t) \sim e^{-t^2/2\sigma^2} \quad (4.1a)$$

$$s_{II}^C(t) \sim te^{-t^2/2\sigma^2}. \quad (4.1b)$$

Linear combinations of these waveforms have been used by some authors to describe essential characteristics of QRS complexes [1], [16], [17]. QRS waveforms with mono- and biphasic configurations can be modelled. The width is easily controlled by σ .

We have used the reduction in signal-to-noise ratio that results from mismatching as our measure of performance. We use the definition

$$\text{SNR}(g(k)) = 10 \cdot 10 \log \frac{\max_k \left\{ \sum_l s(l) g(k-l) \right\}^2}{\int_{-1/2}^{1/2} |G(v)|^2 R_n(v) dv} \quad (4.2)$$

where $g(k)$ is the impulse response of any receiving filter, and $s(k)$ is the input waveform under study, now not necessarily in the class given by (2.2). The maximum SNR is obtained when $g(k)$ is matched to $s(k)$, i.e.

$$\text{SNR}_{\max} = 10 \cdot 10 \log \int_{-1/2}^{1/2} \frac{|S(v)|^2}{R_n(v)} dv. \quad (4.3)$$

In our case the signal $s(k)$ is obtained by sampling one of the continuous time signals, given by (4.1)

$$s(k) = s_i^c \left(\frac{k-\epsilon}{f_s} \right) \quad (4.4)$$

where $|\epsilon| \leq 1/2$. The filter $g(k)$ is in our case $g_{KL}(k, T_i)$, given by (3.12). For the corresponding reduction in SNR, given by the difference between (4.2) and (4.3), we use the notation $R(K, L, \epsilon, T_i)$. If, for given K, L and ϵ , we could choose R_i such that $R(K, L, \epsilon, T_i) = 0$ (i.e. no mismatching), the filter $g_{KL}(k, T_i)$ would be matched to the input in the sense given by Sections II and III (unknown parameter T_i). Thus for each K and L and each input signal under study the optimum T_i is used, i.e. we only use the maximum of $R(K, L, \epsilon, T_i)$ over T_i , which we denote

$$R(K, L, \epsilon) = 10 \cdot 10 \log \max_{T_i} \frac{\max_k \left\{ \sum_l s(l) g_{KL}(k-l, T_i) \right\}^2}{\int_{-1/2}^{1/2} |G_{KL}(v, T_i)|^2 R_n(v) dv} \frac{1/2}{\int_{-1/2}^{1/2} \frac{|S(v)|^2}{R_n(v)} dv} dv \quad (4.5)$$

where $s(k)$ now is given by (4.4). The maximal reduction in SNR due to the choice of arrival time of the input signal is given by

$$R_{\max}(K, L) = \max_{|\epsilon| \leq 1/2} R(K, L, \epsilon) \quad (4.6)$$

which is our measure of mismatching. Even if this quantity is zero, another set of samples from the continuous-time waveform (another ϵ)

will degrade the performance of the KLI-filter, i.e. an extra error in the arrival time will occur. We measure this error as the difference

$$\Delta R(K,L) = \min_{|\epsilon| \leq 1/2} R(K,L,\epsilon) - R_{\max}(K,L). \quad (4.7)$$

Fig. 7 shows $R_{\max}(K,L)$ ¹⁾ for various σ and for the three noise spectra of Fig. 4. It is evident that the case when $K=1, L=0$ does not need further consideration. This is also indicated in Fig. 5, where many of the cases with $L=0$ are not similar to QRS complexes. For small σ ("normal" QRS width) and the coloured noise situation (Fig. 7b and c) the best performance is seen for $s_I^C(t)$. The reason is that the receiver is matched to signals which are essentially monophasic (see Fig. 5 for small I_1). The antisymmetric signal $s_{II}^C(t)$ almost represents a "worst case" for the present filter structure, since the filter essentially "sees the main peak" of the input signal.

In the cases of Fig. 7, the reduction $\Delta R(K,L)$ was always less than 1.5 dB for signal I and 2.6 dB for signal II. Disregarding the case $K=1, L=0$, $\Delta R(K,L)$ was less than 1 dB for all cases. As expected $\Delta R(K,L)$ decreases with increasing σ . For $\sigma \geq 2$, $\Delta R(K,L)$ was less than 0.5 dB.

With the KLI-filter and our assumption of a two-sided a priori probability density for the amplitude, the estimator can "follow" changes in width and shape. Such changes may be due to positional changes of the patient or to aberrant conduction and VES.

¹⁾ The optimization in (4.6) and (4.7) is approximately performed by varying ϵ in steps of $1/30$. Since the signals (4.1) are not band-limited, aliasing will occur in (4.4). This means that $SNR_{\max}$ in (4.3) will depend on ϵ . The greatest variation was only 0.05 dB.

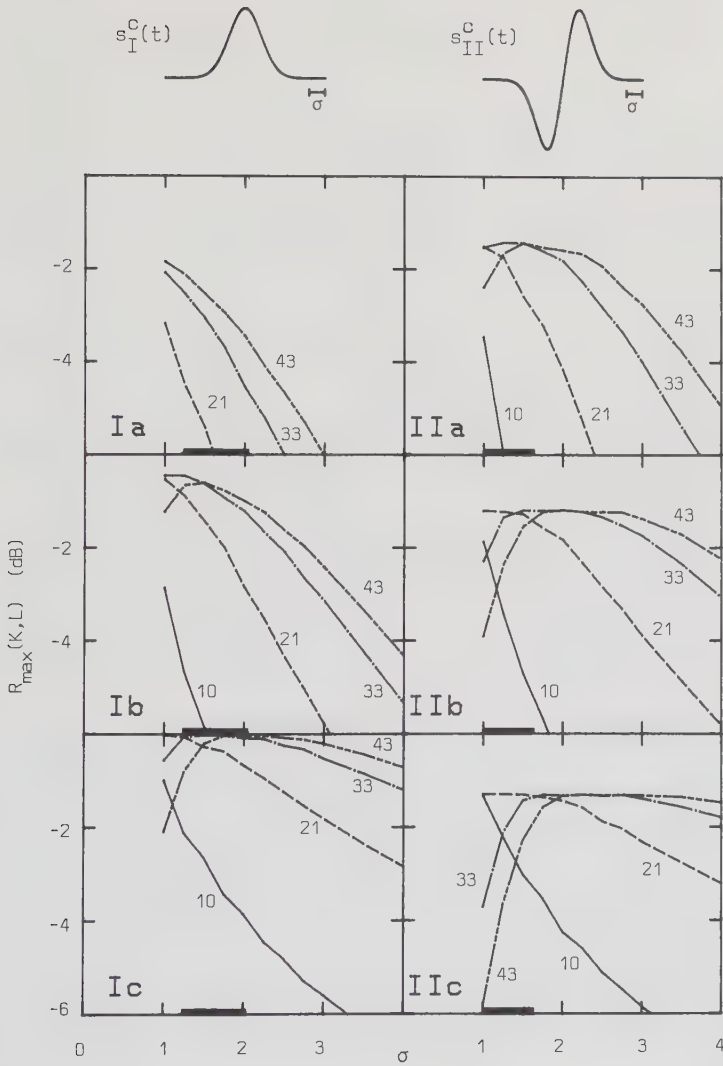


Fig. 7. Reduction in SNR due to mismatching, $R_{\max}(K,L)$, using the filter $g_{KL}(k,T)$ for the input signals $s_I^C(t)$ or $s_{II}^C(t)$ in the three different noise situations. $R_{\max}(K,L)$ is shown as a function of the width parameter σ . Diagrams Ia and IIa corresponds to the noise spectrum with $a = b$, Ib and IIf to $a = 3/4$, $b = 1/4$ and Ic and IIc to $a = 0.9999$, $b = 0$. The two numbers at each curve are the filter parameters KL . The range of normal QRS width (0.06 sec - 0.1 sec) is indicated by a broader line along the σ axis.

IV.3 Delimitation of the observation interval

The estimator works in an observation interval of known length, N . When applying the estimator to real ECGs, an observation interval must first be defined. In Section II it was also assumed that all the waveforms in the interval were completely contained in it. One approach to defining such an interval is to let two "significant" waveforms delimit it. If these waveforms, which we denote "type-events", are actually QRS complexes, an interval has been delimited which completely contains all QRS complexes in between (a certain time must elapse between two consecutive QRS complexes). To determine a type-event, the estimator is used in the same way as before (cf. Sections II and III) but the procedure is finished after estimating the first arrival time, which thus becomes the location of the new type-event. Now, the observation interval is of length N' ($>N$) and starts at the "distance" D from the last detected type-event. The choice of this "primary" interval length N' depends on the environment in which the detector should work. If the interval is too short it may contain nothing; if it is too long the signal properties may have changed quite drastically. In real-time applications, e.g. in a coronary care unit the length N' is upwards limited by the size of the data buffer, or by the maximum acceptable delay without serious medical consequences.

From the type-events one could obtain an estimate of the statistical parameters β_1 , β_2 and τ_1 , τ_2 which define the a priori probabilities $p(B_i)$ and $\Pr(I_i)$, respectively. This kind of estimation is further discussed in part II of the thesis. With this structure, the detector adapts quickly to the prevailing waveform, provided that liberal acceptance criteria are used in the selection of the type-events. After having determined the type-event the QRS detector works in a "non-causal" way. Decisions based on signal properties of the past as well as the future seem more reliable to us than decisions based only on properties of the past. The delimitation and estimation processes are summarized in Fig. 8.

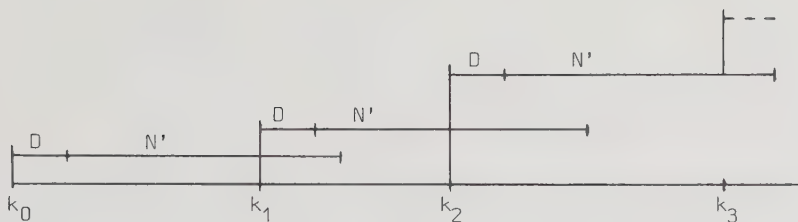


Fig. 8. Delimitation of the observation interval. The time k_0 denotes the arrival time of a type-event. The signal is inspected in the primary interval of length N' starting at (k_0+D) . The next type-event is found at k_1 . The detector now uses (k_0+D, k_1-D) as an observation interval for the estimation procedure. Next a new primary interval of length N' starts at (k_1+D) and the type-event is found at k_2 and so on.

APPENDIX A: CALCULATION OF THE FUNCTION $\hat{B}_i(\theta_i, T_i)$

Let us denote each term in the first sum on the right hand side of (2.17) with $U(\theta_i, B_i, T_i)$. The value of this function is the contribution to the log-likelihood function for one "pulse" with the arrival time θ_i , amplitude B_i and width T_i . We shall maximize $U(\theta_i, B_i, T_i)$ with respect to B_i , resulting in a function $\hat{B}_i(\theta_i, T_i)$.

The expression $\ln X_B(|B_i|)$ is either zero or negative infinity, and thus $\hat{B}_i(\theta_i, T_i)$ should be such that $\beta_1 \leq |\hat{B}_i(\theta_i, T_i)| \leq \beta_2$. Completing the square in $U(\theta_i, B_i, T_i)$ yields

$$U(\theta_i, B_i, T_i) = \frac{d^2(T_i)}{2} [x^2 - (x - B_i)^2 + \ln X_B(|B_i|)] \quad (A.1)$$

where

$$x = x(\theta_i, T_i) = \frac{y(\theta_i) - y(\theta_i - T_i)}{1 - \rho(T_i)} \quad (A.2)$$

The maximum over B_i is given by

$$U(\theta_i, \hat{B}_i(\theta_i, T_i), T_i) = \frac{d^2(T_i)}{2} \cdot \begin{cases} 2|x| & \beta_1 - \beta_1^2 & |x| < \beta_1 \\ |x|^2 & \beta_1 \leq |x| \leq \beta_2 \\ 2|x| & \beta_2 - \beta_2^2 & |x| > \beta_2 \end{cases} \quad (A.3)$$

with the corresponding function

$$\hat{B}_i(\theta_i, T_i) = \begin{cases} \beta_1 \operatorname{sign}(x) & |x| < \beta_1 \\ x & \beta_1 \leq |x| \leq \beta_2 \\ \beta_2 \operatorname{sign}(x) & |x| > \beta_2 \end{cases} \quad (A.4)$$

We rewrite (A.3) as

$$U(\theta_i, \hat{B}_i(\theta_i, T_i), T_i) = d^2(T_i) \left[\beta_1 F(x) - \frac{\beta_1^2}{2} \right] \quad (A.5)$$

where we have introduced the function

$$F(x) = \begin{cases} |x| & |x| < \beta_1 \\ \frac{|x|^2 + \beta_1^2}{2\beta_1} & \beta_1 \leq |x| \leq \beta_2 \\ \frac{2|x|\beta_2 + \beta_1^2 - \beta_2^2}{2\beta_1} & |x| > \beta_2 \end{cases} \quad (A.6)$$

Replacing each term of the first sum in (2.17) with (A.5), yields the log-likelihood function (2.18). The function $F(x)$ is shown in Fig. 9.

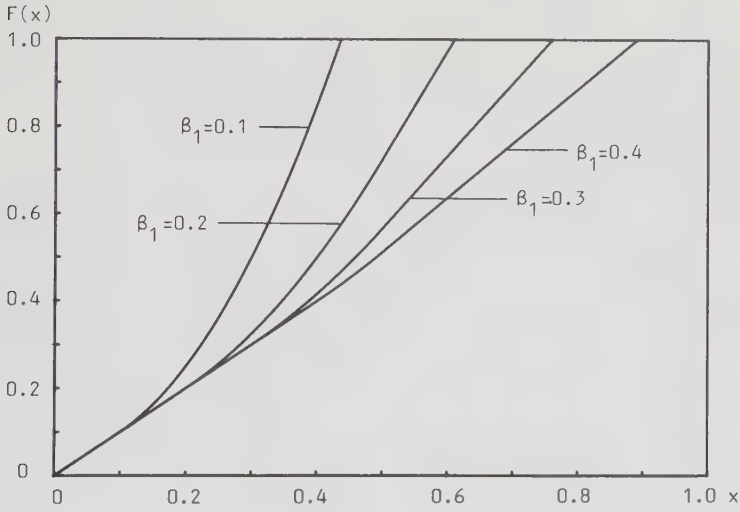


Fig. 9. The function $F(x)$ for $\beta_2 = 0.5$.

APPENDIX B: THREE EXAMPLES OF APPROXIMATE MAP ESTIMATION

In order to reduce the computational load, we have introduced an approximate way of finding the estimates of the arrival times. The use of this estimation procedure is illustrated by means of three examples.

Example 1

Let us consider the case when $n=3$ and $p_i = \frac{1}{4}$, $i = 0,1,2,3$ (equal a priori probabilities for q). The function $M(k)$ used in (2.25) is shown in Fig. 10(a). The parameter D is shown in the same time scale. The width (above threshold $\alpha_1(\theta_1)$) of each peak in the output signal is smaller than D . Using the approximate MAP estimation described by (2.25)-(2.26) we get

$$\left\{ \begin{array}{l} \Delta_0 = 0 \\ \Delta_1 \sim M(k_2) - 0.40 = 0.55 \quad ; \quad \tilde{\theta}_1 = k_2 \\ \Delta_2 \sim M(k_3) - 0.40 = 0.50 \quad ; \quad \tilde{\theta}_2 = k_3 \\ \Delta_3 \sim M(k_4) - 0.40 = -0.10 \end{array} \right. \quad (B.1)$$

From (2.24) we get

$$\left\{ \begin{array}{l} V_0 = 0 \\ V_1 \sim 0.55 \\ V_2 \sim 1.05 \\ V_3 < V_2 \end{array} \right. \quad (B.2)$$

and thus

$$(\tilde{q}, \tilde{\theta}) = (\hat{q}, \hat{\theta}) = (2, k_2, k_3). \quad (B.3)$$

The strategy is to pick the maxima in turn starting with the largest one while excluding a segment of length D around each picked maximum. This scheme is continued until no further maximum exceeding $\beta_1/2$ exists. In this case the result of the approximate strategy is *identical* to the optimum one.

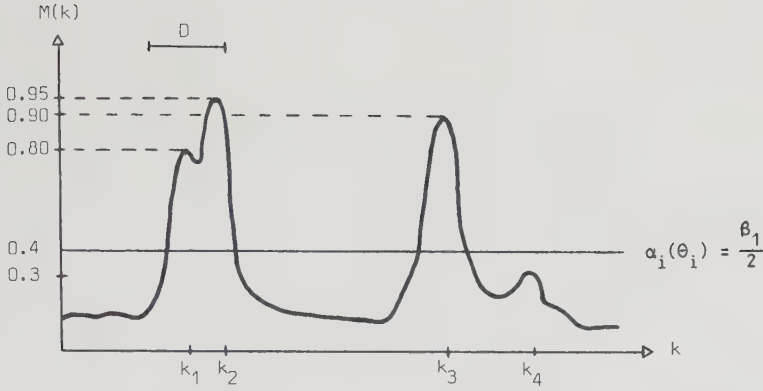


Fig. 10(a). The function $M(k)$ used in (2.25), for simplicity plotted as a continuous-time signal, and the threshold $\alpha_i(\theta_i)$ when equal a priori probabilities p_i .

Example 2

In this example, the a priori probabilities for q are unequal, $(p_0, p_1, p_2, p_3) = (1/8, 1/4, 1/2, 1/8)$, but with n still equal to 3 and $d_0^2 \beta_1 = 10 \cdot \ln 2$. The output is shown in Fig. 10(b).

Calculating (2.24) we obtain

$$\Delta_0 = 0$$

$$\Delta_1 \sim M(k_3) - \left(0.4 + \frac{\ln \frac{4}{8}}{d_0^2 \beta_1} \right) = 0.55$$

$$\Delta_2 \sim M(k_1) - \left(0.4 + \frac{\ln \frac{2}{4}}{d_0^2 \beta_1} \right) = 0.45 \quad (\text{B.4})$$

$$\Delta_3 \sim M(k_2) - \left(0.4 + \frac{\ln \frac{8}{2}}{d_0^2 \beta_1} \right) = 0.15$$

and the estimate is

$$(\tilde{q}, \tilde{\theta}) = (\hat{q}, \hat{\theta}) = (2, k_3, k_1). \quad (B.5)$$

The strategy is not so simple now as in Example 1, where the non-excluded peaks were only compared to a constant threshold. In the general case we must calculate all the quantities V_q and choose the largest. Also in this case, the approximate strategy yields the same estimates as the optimum one.

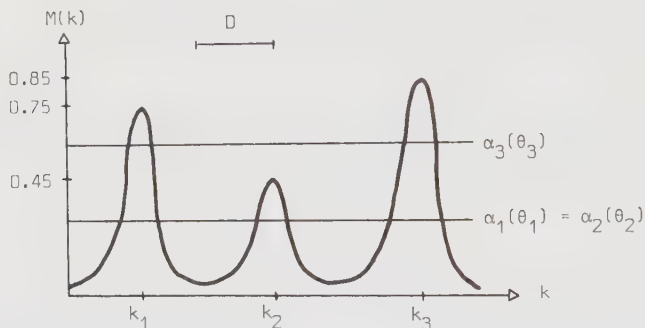


Fig. 10(b). The function $M(k)$ used in (2.25) and the threshold $\alpha_i(\theta_i)$ when unequal p_i .

Example 3

We consider $n=6$ and $p_i = \frac{1}{7}$ for $i = 0, 1, \dots, 6$ but in this case the output signal contains a wide peak as shown in Fig. 10(c).

We sum up (2.24) to yield the "log-likelihood function" V_q ,

$$V_q = d_0^2 \beta_1 \sum_{i=1}^q \max_{\theta_i} [M(\theta_i) - \alpha_i(\theta_i)]. \quad (B.6)$$

Then for $d_0^2 \beta_1 = 1$ we get

$$\begin{cases} V_0 = 0 \\ V_1 = M(k_2) - 0.4 = 0.5 \\ V_2 = V_1 + M(k_4) - 0.4 = 0.85 \\ V_3 < V_2 \end{cases} \quad (B.7)$$

and

$$(\tilde{q}, \tilde{\theta}) = (2, k_2, k_4). \quad (B.8)$$

In this case the result of the approximate strategy differs from the optimum one. Eqn. (2.20) yields for

$$\begin{aligned} V(\underline{r}, 0) &= 0 \\ V(\underline{r}, 1, \theta_1) &= M(k_2) - 0.4 = 0.5 \\ V(\underline{r}, 2, \theta_1, \theta_2) &= M(k_1) + M(k_3) - 2 \cdot 0.4 = 0.8 > V(\underline{r}, 1, \theta_1) \\ V(\underline{r}, 3, \theta_1, \theta_2, \theta_3) &= M(k_1) + M(k_3) + M(k_4) - 3 \cdot 0.4 = 1.0 > V(\underline{r}, 2, \theta_1, \theta_2) \\ V(\underline{r}, 4, \theta_1, \theta_2, \theta_3, \theta_4) &< V(\underline{r}, 3, \theta_1, \theta_2, \theta_3) \end{aligned} \quad (B.9)$$

and

$$(\hat{q}, \hat{\theta}) = (3, k_1, k_3, k_4). \quad (B.10)$$

Note that k_1 and k_3 are not obtained from the peaks in the output signal.

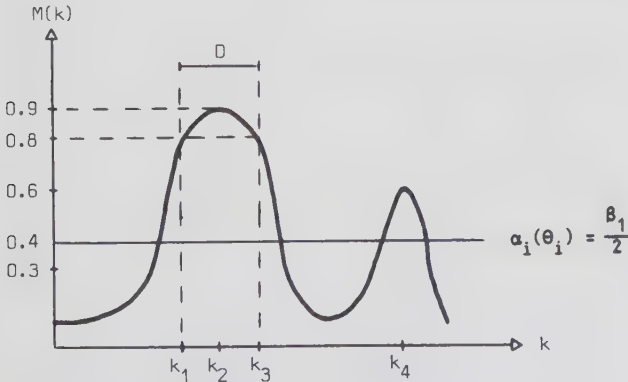


Fig. 10(c). The function $M(k)$ used in (2.25) and the threshold $\alpha_i(\theta_i)$ when a wide peak occurs.

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ADAPTIVE QRS DETECTION: A STUDY OF PERFORMANCE

PART II

ABSTRACT

QRS detectors are often evaluated in terms of statistical measures, e.g. rate of false and true detections. These measures are usually calculated for a detector with fixed parameter values. In this part, detector properties are studied by varying those parameters which seem to have the most profound effect on performance. By applying techniques known from optimal estimation to a stochastic model for the ECG, two different detectors have been derived. The detector structures are described in detail. In order to study and compare the performance of the two detectors a material of "difficult" ECGs has been collected, containing various "artefacts" of physiological and electrical/mechanical origin. The performance on this material is presented in terms of receiver-operating characteristics (ROCs) and several examples are discussed.

I. INTRODUCTION

The accuracy of the QRS detector is by now recognized as a crucial factor for the over-all performance of computer-based arrhythmia monitoring. A variety of QRS detectors have been presented in the literature, usually with emphasis on algorithmic development. Before implementation in a clinical setting, the detector has to be evaluated on a sample of ECGs. While some consider the performance of the entire analysis system as a measure of the performance of the detector, e.g. [1], most detectors are evaluated in terms of falsely detected and missed QRS complexes [2]-[5]. Until recently the lack of a unified ECG-database suitable for QRS detector evaluation has made different workers collect their own materials. Since the signal quality and the number of premature ventricular complexes (PVCs) may vary heavily from material to material, a comparison of the obtained results is not meaningful. A low error rate of the detector may just as well reflect a non-provocative material as an acceptable performance.

The purpose of performance evaluation of QRS detectors may be considered in two parts; to gather insight into how the mechanisms of the detector behave for different choices of parameter values, and, for the detector with fixed parameters, to calculate statistical measures describing the over-all performance. The latter part is a "black-box" approach since it yields little information for the further refinement of the detector. This kind of evaluation, which is the predominant one in the literature, is the final step before implementation of the detector in clinical routine, and requires a large, representative set of ECG-data. While the former kind of evaluation also may utilize statistical performance measures, these measures are calculated in order to describe e.g. the sensitivity of the detector for different parameter values, and must not be compared with the over-all performance. Here, the receiver operating characteristic (ROC) is a useful tool in presenting the performance of the detectors, if these are evaluated on the same material. The ROC displays the true detection rate versus the false detection rate for different values of a certain parameter, and may permit the choice of a suitable operating point.

A means for studying the performance is to employ simulated ECGs. The generation of such signals renders it possible to easily investigate e.g. the robustness of the detector to different kinds of noise and artefacts. In that case, curves in the ROC may be calculated for different signal-to-

Adaptive QRS detection: performance

noise ratios (SNRs), while for real ECGs only one curve may be obtained. Also, with simulated signals the influence of changing QRS morphology as well as the rejection of tall T-waves may be studied in detail. Modelling of the QRS complex has been treated within various contexts [6]-[8].

A mathematical model for the ECG has been proposed in part I, in which the QRS complexes have random amplitudes, widths and arrival times. Also in each observation interval the number of complexes is unknown. From the observed signal, the detector or rather the estimator, tries to find those parameter values that give the best fit to the model. Since the computational effort associated with the maximum-a-posteriori (MAP) estimator is large, a suboptimal estimator was introduced (SMAP). In this part, an additional simplification is introduced in which the estimator uses only peaks in the filtered signal (PMAP). This yields a drastic decrease in computation time. The structure of the detectors is described in detail, as well as the estimation of the statistical parameters. The performance is investigated on a material of "difficult" ECGs, and is presented in terms of ROCs. To shed light upon these results, several examples are discussed which show the behaviour of the detectors. Based on these results, various improvements of the ECG model are suggested.

II. ADAPTIVE QRS DETECTION BASED ON MAP ESTIMATION

II.1 Signal model

The ECG signal is modelled as a discrete-time stochastic process; in an observation interval, $1 \leq k \leq N$, an unknown number, q , of pulse shaped waveforms occur which are corrupted by stationary, zero-mean, Gaussian noise,

$$r(k) = \begin{cases} \sum_{i=1}^q B_i s(k - \theta_i, T_i) + n(k) & 1 \leq q \leq n \\ n(k) & q = 0. \end{cases} \quad (2.1)$$

The waveforms $s(k, T_i)$ belong to a known class of signals, but with unknown arrival time θ_i , amplitude B_i and width T_i , and has a duration less than D . Furthermore, $s(k, T_i)$ is composed of two equal waveforms $v(k)$, separated by the distance T_i ,

$$s(k, T_i) = \begin{cases} v(k) - v(k + T_i) & 0 \leq k \leq D-1 \\ 0 & \text{otherwise.} \end{cases} \quad (2.2)$$

The parameters q , θ_i and T_i are discrete-valued random variables and B_i is continuous-valued. The noise $n(k)$ is white, but may be generalized to the coloured situation, see p. 43ff.

Due to the refractory period of the heart, two QRS complexes cannot occur simultaneously, and thus the arrival times $\theta_1, \dots, \theta_q$ are assumed to be separated by at least the distance D , i.e.

$$|\theta_i - \theta_j| \geq D \quad \text{for} \quad i \neq j. \quad (2.3a)$$

In the model this implies that the waveforms are non-overlapping. To guarantee that all waveforms are completely contained in the observation interval $1 \leq k \leq N$,

$$1 \leq \theta_i \leq N - (D - 1) \quad i = 1, \dots, n. \quad (2.3b)$$

Furthermore, we require that q fulfils

$$q \leq n \leq \left\lfloor \frac{N-D}{2D-1} + 1 \right\rfloor, \quad (2.4)$$

where $\lfloor \cdot \rfloor$ denotes the integer part. This allows us to assume that q and $\underline{\Theta}$ are independent. All random variables are independent, apart from an interdependence between the components in $\underline{\Theta}$. Then, the joint probability density can be written

$$p(q, \underline{\Theta}, \underline{B}, \underline{I}) = \Pr(q) \Pr(\underline{\Theta}) \prod_{i=1}^n p(B_i) \Pr(I_i). \quad (2.5)$$

The probability for q waveforms is

$$\Pr(q) = \begin{cases} p_q & q = 0, 1, \dots, n \\ 0 & \text{otherwise.} \end{cases} \quad (2.6)$$

The joint probability $\Pr(\underline{\Theta})$ is uniform over those $\underline{\Theta}$ which fulfil (2.3). The probability density $p(B_i)$ is double-sided uniform in the intervals $[-\beta_2, -\beta_1]$ or $[\beta_1, \beta_2]$, where $0 < \beta_1 < \beta_2$, and $\Pr(I_i)$ is uniform in the interval $[\tau_1, \tau_2]$, where τ_1 and τ_2 are positive integers.

II.2 Detector structure

Based on the observed signal $r(k)$ in (2.1), the structure of SMAP is derived by maximizing the log-likelihood function

$$\begin{aligned} V\left(r(k), q, \underline{\Theta}, \underline{B}, \underline{I}\right) &= \frac{2}{N_0} \sum_{k=1}^N r(k) \sum_{i=1}^q B_i s(k-\Theta_i, I_i) \\ &- \frac{1}{N_0} \sum_{k=1}^N \left(\sum_{i=1}^q B_i s(k-\Theta_i, I_i) \right)^2 + \ln p(q, \underline{\Theta}, \underline{B}, \underline{I}) \end{aligned} \quad (2.7)$$

with respect to the random variables to be estimated, in our case $(q, \underline{\Theta}, \underline{B}, \underline{I})$. The variance of the white noise is $N_0/2$. We will here give the equations which yield the estimates; a detailed derivation is found in part I, Sec. II.

Due to the first term in (2.7), the observed signal $r(k)$ is first passed through a linear time-invariant filter ("matched" filter). In this study, filters are considered which have an impulse response defined by two integer parameters K and L ,

$$h_{KL}(k) = Z^{-1} \left\{ (1-z^{-K}) (1+z^{-1})^L \right\} \quad (2.8)$$

where $Z^{-1}\{\cdot\}$ is the inverse Z -transform. Since the noise is white, we have that $v(-k) \sim h_{KL}(k)$. The frequency function for $h_{KL}(k)$ is shown on p. 9 for some values of K and L .

The maximization of (2.7) with respect to B_i and T_i yields that the filtered signal

$$y(\theta_i) = \sum_k r(k) h_{KL}(\theta_i - k) \quad (2.9)$$

is transformed by a non-linear scheme. The resulting signal $M(\theta_i)$ is defined by the following relation

$$M(\theta_i) = \max_{\tau_1 \leq T_i \leq \tau_2} \left[\left(1 - \rho(T_i) \right) F(x(\theta_i, T_i)) + \rho(T_i) \frac{\beta_1}{2} \right] \quad (2.10)$$

where the function $F(x(\theta_i, T_i))$ is given on p. 39, and

$$x(\theta_i, T_i) = \frac{y(\theta_i) - y(\theta_i - T_i)}{1 - \rho(T_i)}. \quad (2.11)$$

The function $\rho(T_i)$ is the normalized correlation between $v(k)$ and $v(k+T_i)$, and the energy of $v(k)$ is denoted with E_v . Note that for a given value of i and θ_i , the width estimate $\tilde{T}_i$ is that T_i which maximizes the right hand side in (2.10).

In order to determine the estimates $\tilde{q}, \tilde{\theta}_1, \dots, \tilde{\theta}_q$, the sum V_q is calculated,

$$V_q = \begin{cases} \sum_{i=1}^q \Delta_i & 1 \leq q \leq n \\ 0 & q = 0 \end{cases} \quad (2.12)$$

where

$$\Delta_i = \max_{\theta_i} [M(\theta_i) - \alpha_i(\theta_i)] \quad (2.13)$$

and

$$\alpha_i(\theta_i) = \frac{\beta_1}{2} + \frac{N_0}{4E_v\beta_1} \ln \frac{p_{i-1}}{p_i} - \ln \chi(\tilde{\theta}_1, \tilde{\theta}_2, \dots, \theta_i) \quad (2.14)$$

where $\chi(\cdot)$ is equal to one if $\tilde{\theta}_1, \dots, \theta_i$ fulfils (2.3), otherwise zero. The estimate $\tilde{q}$ is obtained by choosing that q for which the sum V_q is largest. The estimates $\tilde{\theta}_1, \dots, \tilde{\theta}_{\tilde{q}}$ are obtained successively, such that an estimate is yielded by that θ_i which maximizes (2.13). If $p_{i-1} \geq p_i$, then the equations (2.12)-(2.14) could be restated by the following rule:

Choose the maxima in the signal $M(\theta_i)$ in turn starting with the largest one and ignoring such maxima which fall within the distance D from an already picked maximum. The time instance for each picked maximum is denoted with $\tilde{\theta}_i$. This scheme is continued until no further maximum exceeding $\alpha_i(\theta_i)$ exists, or until (2.4) no longer is fulfilled. If no peak in $M(\theta_i)$ exceeds $\alpha_i(\theta_i)$, then $\tilde{q} = 0$.

Note that $\tilde{\theta}_1, \dots, \tilde{\theta}_{\tilde{q}}$ are not necessarily found in temporal order.

If $p_{i-1} \geq p_i$ and the lower limit β_1 in $p(\beta_i)$ is such that

$$\beta_1^2 > \frac{N_0 \max_i \ln \frac{p_{i-1}}{p_i}}{2E_v \min_{T_i} (1-p(T_i))} \quad (2.15)$$

where $\tau_1 \leq T_i \leq \tau_2$ and $i = 1, \dots, n$ (see Appendix), then $F(x(\theta_i, T_i))$ in (2.10) could be replaced by

$$F(x(\theta_i, T_i)) = |x(\theta_i, T_i)|. \quad (2.16)$$

Note that for the important case when p_i is uniform, it is always possible to use (2.16).

Adaptive QRS detection: performance

Even if the estimates $\tilde{q}$ and $\tilde{\Theta}$ are those quantities which are of ultimate interest, we shall need an estimate of B_1 . With our assumption of a double-sided uniform density for B_1 , the amplitude estimate is given by

$$\tilde{B}_1(\theta_i, T_i) = \begin{cases} \beta_1 \operatorname{sign}(x(\theta_i, T_i)) & |x(\theta_i, T_i)| < \beta_1 \\ x(\theta_i, T_i) & \beta_1 < |x(\theta_i, T_i)| < \beta_2 \\ \beta_2 \operatorname{sign}(x(\theta_i, T_i)) & \beta_2 < |x(\theta_i, T_i)|. \end{cases} \quad (2.17)$$

For a given value of i , θ_i and T_i , we obtain the estimate $\tilde{B}_1$.

II.3 Peak-picking for calculation of $M(\theta_i)$

In SMAP the estimates $\tilde{q}$ and $\tilde{\Theta}$ are obtained by comparing $M(\theta_i)$ to the threshold $\alpha_i(\theta_i)$. Since the maximization over T_i must be carried out for each sample in the filtered signal, the calculation of $M(\theta_i)$ is time-consuming. In many situations, the maximum in (2.10) is obtained by simply picking $y(\theta_i)$ and $y(\theta_i - T_i)$ among the peaks and valleys in the signal. Clearly, this peak-picking strategy produces a signal different from (2.10), but yields a substantial decrease in computation time. For sake of clarity, the subindex i is omitted in the remaining part of this subsection.

The simplified detector, PMAP, is defined as SMAP but with the function $M(\theta)$ in (2.10) replaced by $M^P(\theta)$. This new function is obtained from the sequence of peaks and valleys in $y(\theta)$, here denoted with $y^P(\ell; \theta_\ell)$, $\ell = 1, 2, \dots, N_p$, $N_p \leq N$. A peak or valley is found at the point θ if $|y(\theta-1)| < |y(\theta)|$ and $|y(\theta+1)| < |y(\theta)|$. The signal $M^P(\theta)$ is then defined by

$$M^P(\theta) = \begin{cases} \max_{1 \leq j \leq j_1} \left[\left(1 - \rho(T_{\ell, j}) \right) F(x^P(\ell, j)) + \rho(T_{\ell, j}) \frac{\beta_1}{2} \right] & \begin{aligned} \theta &= \theta_\ell, \\ \ell &= 2, 3, \dots, N_p, \\ \tilde{T}_\ell &\text{ exists} \end{aligned} \\ 0 & \text{otherwise} \end{cases} \quad (2.18)$$

where

$$x^P(\ell, j) = \frac{y^P(\ell; \theta_\ell) - y^P(\ell-j; \theta_{\ell-j})}{1 - \rho(\tau_{\ell, j})}$$

and

$$j_1 = \min(\ell-1, \tau_2).$$

The maximization in (2.18) is subject to the constraint that

$$\tau_{\ell, j} = \theta_\ell - \theta_{\ell-j} \quad (2.19)$$

is such that $\tau_1 \leq \tau_{\ell, j} \leq \tau_2$. If no index j is found for which $\tau_1 \leq \tau_{\ell, j} \leq \tau_2$, then a width estimate $\tilde{\tau}_\ell$ does not exist and thus $M^P(\theta)$ is equal to zero. Otherwise, the estimate $\tilde{\tau}_\ell$ is given by (2.19) for that index j which maximizes (2.18).

II.4 Estimation of statistical parameters

The processing to be performed on the observed ECG signal $r(k)$ has been divided into two different phases:

- I. Delimitation of the observation interval by means of "significant" waveforms (type-events) followed by
- II. Estimation of the number of waveforms and their arrival times in the delimited interval.

In both phases, the same detector structure is used, but with different parameter values. The number of waveforms is in phase I either equal to zero or one. After the detector has terminated phase II, a new observation interval is delimited and so on.

So far, the detector structure used for both phase I and II has been described. We will now consider the problem of estimating the statistical parameters β_1 , β_2 , τ_1 and τ_2 which define $p(B_i)$ and $\Pr(I_i)$. Since the QRS morphology often exhibits a short-term variability, the statistical parameters should be estimated from the prevailing morphology rather than preset. Such estimates can be obtained by maximizing the log likelihood

function (2.7) with respect to the statistical parameters. However, a difficulty arises when using this approach since $\tilde{q}$, $\tilde{\underline{Q}}$, $\tilde{\underline{B}}$ and $\tilde{\underline{I}}$ are required for obtaining the estimates and vice versa [9]. An iterative procedure similar to the one suggested in [10] may be introduced to obtain both kind of estimates, but at the expense of a substantially increased amount of computations.

A simple method is to use already available estimates of B and I for estimating the statistical parameters. In general, if $Z_1, Z_2, \dots, Z_p$ are independent random variables, each of which either is distributed uniformly over $[\Delta_1, \Delta_2]$ or double-sided uniformly over $[-\Delta_2, -\Delta_1]$ and $[\Delta_1, \Delta_2]$, $0 \leq \Delta_1 \leq \Delta_2$, then the joint maximum-likelihood (ML) estimates of Δ_1 and Δ_2 are given by

$$\hat{\Delta}_1 = \min(|z_1|, \dots, |z_p|) \quad (2.20a)$$

and

$$\hat{\Delta}_2 = \max(|z_1|, \dots, |z_p|). \quad (2.20b)$$

In order to compensate for the bias in these estimates, i.e.

$$E[\hat{\Delta}_i] = \Delta_i + (\Delta_j - \Delta_i) \cdot \frac{1}{p+1} \quad i, j = 1, 2; \quad i \neq j \quad (2.21)$$

it is useful to introduce an additive or multiplicative parameter on the right hand side of (2.20).

By tacitly assuming that $p(\tilde{B}_i)$ is identical to $p(B_i)$ and $\Pr(\tilde{I}_i)$ to $\Pr(I_i)$, estimates of β_1 , β_2 , τ_1 and τ_2 can be obtained by (2.20). By using only the p most recent estimates of B_i or I_i in the ML-estimator, the detector adapts itself more or less fast to the prevailing morphology. Below, estimates are obtained with $p=2$ which thus yields a short-term adaptivity of the detector. In order to find the type-event in phase I, an observation interval of length N^I is processed by using

$$\hat{\beta}_2^I = \lambda_2^I \max(|\tilde{B}_{-1}|, |\tilde{B}_0|) \quad \lambda_2^I > 1 \quad (2.22a)$$

$$\hat{\beta}_1^I = \lambda_1^I \hat{\beta}_2^I \quad 0 < \lambda_1^I < 1 \quad (2.22b)$$

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where $\tilde{B}_{-1}$ and $\tilde{B}_0$ denotes the amplitude estimates for the two most recently detected type-events. An upper index I is added to those parameters that belong to phase I. The estimate $\hat{\beta}_1^I$ is related to $\hat{\beta}_2^I$ in order to reduce the risk of choosing the type-event among noise deflections. By means of the parameters λ_1^I and λ_2^I it is possible to control the sensitivity of the detector to abrupt changes in amplitude. Note that λ_2^I must be greater than one, otherwise the upper limit $\hat{\beta}_2^I$ is decreasing or possibly unchanged, from phase to phase. The choice of τ_1^I and τ_2^I is found to be less critical, and are therefore simply considered to be fixed-valued.

In the second phase, the type-events which delimit the observation interval are used to determine estimates of the statistical parameters. The amplitude parameters are updated by

$$\hat{\beta}_2 = \lambda_2 \max(|\tilde{B}_0|, |\tilde{B}_{q+1}|) \quad \lambda_2 > 1 \quad (2.23a)$$

$$\hat{\beta}_1 = \lambda_1 \min(|\tilde{B}_0|, |\tilde{B}_{q+1}|) \quad \lambda_1 > 0. \quad (2.23b)$$

For convenience, the amplitude estimate of the type-event found in phase I is denoted with $\tilde{B}_{q+1}$. (This is somewhat improper since $\hat{\beta}_1$ actually is determined before q). The minimum is used in (2.23b) since the estimates $\tilde{B}_0$ and $\tilde{B}_{q+1}$ "surround" the observation interval, and should thus be more reliable than those used in (2.22a). If the inequality (2.15) holds, an estimate of β_2 is not required in the detector. Note that the estimate $\hat{\beta}_2^I$ is still required even if (2.15) is fulfilled, since the amplitude estimates of the type-events are used in phase II. The width parameters are estimated by means of the relations

$$\hat{\tau}_2 = \max(\tilde{T}_0, \tilde{T}_{q+1}) + \gamma, \quad \hat{\tau}_2 \leq \tau_2^I \quad (2.24a)$$

$$\hat{\tau}_1 = \min(\tilde{T}_0, \tilde{T}_{q+1}) - \gamma, \quad \hat{\tau}_1 \geq \tau_1^I \quad (2.24b)$$

where γ is a positive integer.

If no type-event is found, a new observation interval N^I is analysed. Since this may be due to a sudden change in QRS amplitude, the most recent estimates $\hat{\beta}_1^I$ and $\hat{\beta}_2^I$ are decreased exponentially in each new observation interval until a waveform is found.

II.5 Choice of parameter values

The performance of the detectors is studied by observing the influence on different statistical measures when varying different parameter values. Due to the large number of parameters, one is forced to restrict the study to only investigating those parameters which affect the performance most profoundly. In Table I, parameters with fixed values are shown. Note that with a sampling rate $f_s=100\text{Hz}$, $D=20$ corresponds to a maximal heart rate of 300 beats/minute. The choice of λ_2^I yields a moderately fast return to the "true" QRS amplitude.

Detector Parameter	D	N^I	λ_2^I	λ_1^I	τ_1^I	τ_2^I
Value	20	300	1.3	0.35	2	10

Table I. Fixed parameter values used in detectors SMAP and PMAP.

The parameter λ_1 is crucial since it affects the threshold $\alpha_1(\theta_1)$ in the second phase. Also, the influence of the width parameter γ is investigated, as well as some combinations of the filter parameters K and L . In general, the calculation of the correlation function $\rho(T_i)$ depends on the noise situation. Since the structure of the noise is unknown for real ECGs, $\rho(T_i)$ is set to zero. The calculation of $\rho(T_i)$ for different noise spectra of lowpass character is seen to support this assumption, cf. p. 48. The a priori probability for the number of waveforms is uniform in each observation interval, i.e. in Phase I $p_0 = p_1 = \frac{1}{2}$ and in Phase II $p_q = \frac{1}{n+1}$, $q = 0, 1, \dots, n$.

III. PERFORMANCE MEASURES

The detection of QRS complexes is here treated as the problem of estimating the number of waveforms and their arrival times in a noisy signal. The performance of any estimator is classically considered by establishing a lower bound on the mean-square error for the estimated parameters. For example, tight such lower bounds have been derived in [11], [12], which are useful in arrival time estimation and valid also for low SNR's. These results apply to the situation in which a single waveform is corrupted with Gaussian noise. Since our model assumes the number of waveforms to be random, a performance analysis is complicated. Presently it seems that no paper has been published, which deals with the accuracy in the joint estimates of q and $\underline{Q}$. We will now discuss some ways to measure the performance of the detector.

An M-ary generalized likelihood ratio test (MGLRT) was used in [13] for resolving a sequence of overlapping echo waveforms. That signal model was similar to ours, cf. (2.1), apart from the width T_i not being modelled. The performance of the derived receiver was calculated by means of computer simulations in terms of probabilities of decision D_i under hypothesis H_j , i.e. $P(D_i|H_j)$, $i = 0, \dots, M-1$. The hypothesis H_i denotes that i echo waveforms occur in the observed signal. Based on $P(D_i|H_j)$, one may calculate the error probability

$$P_E = \sum_{i \neq j} P(D_i|H_j) \quad i, j = 0, 1, \dots, M-1 \quad (3.1)$$

and the probability of detection

$$P_D = 1 - P_E. \quad (3.2)$$

Even if the above performance measures are related to the MGLRT, it is also possible to use (3.1)–(3.2) for our estimators. In this case the decision $D_i = \tilde{q}$, while H_j is the outcome of the random variable q . However, a major disadvantage with the above measures is that even if the decision D_i was correct, this does not guarantee that the estimated arrival times correspond to the true ones.

In this study, we consider the quantities true and false detection rate, which to some degree reflect the accuracy of the arrival time estimates.

Adaptive QRS detection: performance

A waveform is defined as a true detection if the true arrival time θ_i and the estimated one $\tilde{\theta}_i$ are such that [14]

$$\tilde{\theta}_i \geq \theta_i - l_1 \quad \text{and} \quad \tilde{\theta}_i \leq \theta_i + l_2, \quad l_1, l_2 > 0 \quad (3.3)$$

where l_1 and l_2 are timing tolerances. If, we have that $l_1 + l_2 + 1 < D$, only one waveform can occur in the interval $[\theta_i - l_1, \theta_i + l_2]$. Since a positive or negative estimation error ($\tilde{\theta}_i - \theta_i$) here is judged equal, we set $l_1 = l_2 = l$. Along with the above definition (3.3), false and true detection rates are defined by

$$\hat{P}_F = \frac{1}{K} \sum_{i=1}^K \hat{P}_F^{(i)}, \quad \hat{P}_D = \frac{1}{K} \sum_{i=1}^K \hat{P}_D^{(i)} \quad (3.4)$$

respectively, where

$$\hat{P}_F^{(i)} = \frac{N_F^{(i)}}{N_T^{(i)} + N_F^{(i)}}, \quad \hat{P}_D^{(i)} = 1 - \frac{N_M^{(i)}}{N_T^{(i)}}.$$

In (3.4), K is the number of subsets, which e.g. is the number of ECG episodes, the number of classes or the total material ($K=1$). Furthermore, $N_F^{(i)}$ is the number of false detections in subset i , $N_M^{(i)}$ the number of missed QRS complexes and $N_T^{(i)}$ the number of true complexes.

IV. PERFORMANCE ON SIMULATED ECGs

Due to the uncertainty of the human eye to establish an accurate arrival time of the QRS complexes in noisy ECGs, it is in this case difficult to analyze the performance of the detectors. However, to gain some knowledge in such situations, the ECG model described earlier has been employed. In a sense, this model may provide a "worst case", since both QRS amplitude and width can vary drastically from beat to beat.

The simulated ECG is obtained by

$$r(k+j \cdot N) = \begin{cases} \sum_{i=1}^{q_j} B_{ij} s(k-\theta_{ij}, I_{ij}) + n_j(k) & 1 \leq q \leq n \\ n_j(k) & q = 0 \end{cases} \quad \begin{matrix} j = 0, 1, \dots, I_N-1 \\ q = 0 \end{matrix} \quad (4.1)$$

and the a priori probability $p(q, \underline{\theta}, \underline{B}, \underline{I})$ in (2.5). In each observation interval, new outcomes of $(q, \underline{\theta}, \underline{B}, \underline{I})$ and the noise sequence are determined. In this study, we will only consider waveforms corrupted with white noise. With the choice of parameter values described below, the white noise is a reasonable description of the situation when the ECG is disturbed by noise originating from muscular activity (EMG). The SNR is defined by

$$\text{SNR} = 10 \log \frac{2\sigma_B^2 E[\sum_k s^2(k, I)]}{N_0} \quad [\text{dB}] \quad (4.2)$$

where σ_B^2 is the variance of the amplitude B_i ,

$$\sigma_B^2 = \frac{\beta_2^2}{3} \left(1 + \frac{\beta_1}{\beta_2} + \left(\frac{\beta_1}{\beta_2} \right)^2 \right) \quad (4.3)$$

The term $E[\sum_k s^2(k, I)]$ is the expectation over the width I_1 , which is

$$E[\sum_k s^2(k, I)] = \frac{1}{\tau_2 - \tau_1 + 1} \sum_{I=\tau_1}^{\tau_2} \sum_k s^2(k, I) \quad (4.4)$$

The definition of SNR in (4.2) reduces to the common one if amplitude and width are deterministic properties [15].

In order to generate the ECG, the following parameter values have been used in the model: $\beta_1=25$, $\beta_2=50$, $\tau_1=2$, $\tau_2=6$, $N=300$, $D=20$, $(K,L) = (1,2)$, $p_0=0$, $p_i=1/n$, $i = 1, \dots, n$ and $I_N=2400$. The assumption that $p_0=0$ implies that the lengths of the R-R intervals are in the interval $[D, 2N-D]$. Due to computational reasons, the same pseudo-random sequence has been used for calculating the different points in the ROC. The timing tolerance ℓ in (3.3) is chosen to half the maximum length of the waveform $s(k, T_i)$, which is

$$\ell = \lfloor (K+L+\tau_2-1)/2 \rfloor \quad (4.5)$$

An example of a simulated ECG is shown in Fig. 1 for different SNRs.

Receiver-operating characteristics for SMAP and PMAP are plotted in Fig. 2(a)-(b) for two different SNRs. The performance of the detectors has been studied with parameter values given in Table I, apart from that τ_2^T is equal to 6. It is seen that the difference in performance between SMAP and PMAP is small for $\gamma=4$. However, by narrowing the width acceptance ($\gamma=1$) PMAP degrades more in performance than SMAP does. It should be noted that for $\gamma=4$, the estimates of τ_1 and τ_2 always are equal to those used in the model. By using the same value of λ_1 in both detectors, both true and false detection rate are higher in SMAP. Thus, in order to achieve the same detection rate for both detectors, a smaller value of λ_1 should be used in PMAP. Simulations for higher SNRs (e.g. greater than 30dB) shows that the detection rate is almost identical to one.

A simplification of PMAP is to only use adjacent peaks in the calculation of $M^P(\theta_i)$, i.e. for j_1 equal to one in (2.18). Such a procedure has been suggested in [16] and [17]. As the results in Fig. 2(a) indicate, a substantial reduction in performance is obtained. This behaviour is essentially due to notched waveforms being missed by the detector.

Adaptive QRS detection: performance

The computational effort of SMAP is almost independent of the noise level in the signal. Of course, since a noisy signal contains more peaks, this level greatly affects the amount of computations in PMAP. For a moderately disturbed signal ($\text{SNR} \geq 30\text{dB}$), the amount of CPU time required for PMAP is 7-8 times less than for SMAP, whereas for signals with excessive noise, this ratio is reduced to 4.

V. PERFORMANCE ON REAL ECGs

V.1 Materials

A material consisting of 100 4-minute ECG episodes has been collected (sampling rate $f_s = 100\text{Hz}$ and thus the length of the sampling interval is 10ms). This was done by a physician who was unaware of the properties of SMAP and PMAP. The material was divided into two different categories, one containing anomalies with physiological origin and one with electrical/mechanical artefacts. These categories have been divided into six different classes, see Table II. Each class includes 20 ECGs, except for class 3 and 4 which each includes 10 ECGs. Each ECG contained only one of the six anomalies, and each patient contributed at most to two different classes. Approximately one half of the ECGs in class 2 to 6 had irregular rhythm. In class 6, only such ECGs were included which allowed visual definition of the arrival time of the QRS complexes. No training was performed on the collected material. A 15 second "normal" signal portion before each 4-minute ECG was required for detector learning. It should be emphasized that the material does *not* reflect the overall performance of a detector working in clinical routine, but is intended for pointing out weaknesses in the detectors, and accordingly also in the ECG model.

Anomalies with physiological origin	1. Abrupt changes in rhythm, e.g. from sinus rhythm to supraventricular or ventricular tachycardia.
	2. Sporadic or in couple occurring beats with aberrant morphology.
	3. Large P- or T-waves (with an amplitude at least 2/3 of the QRS).
Anomalies with electrical/ mechanical origin	4. Heavy baseline shifts or wandering.
	5. Amplitude variation of the QRS complexes.
	6. High-frequency noise, e.g. muscular disturbances.

Table II. Categories of ECG signals used for study of QRS detector performance.

V.2 Receiver-operating characteristics

The performance of SMAP and PMAP is presented in the ROCs in Fig. 3(a)-(d) for three different filters. The ROCs were computed for the total ECG material. Each curve was computed by varying λ_1 in (2.23b) in steps of $\Delta\lambda_1 = 0.1$ decreasing from $\lambda_{1,\max} = 1.7$, while keeping γ constant. The timing tolerance ℓ was equal to 7 samples.

The filter parameters $(K,L) = (1,2)$ yielded the best performance for both SMAP and PMAP. For this filter it is seen that SMAP is unsensitive for different γ : almost identical detection rates can be obtained for $\gamma = 2, 3$ or 4, while a slight increase in $\hat{P}_F$ is obtained for $\gamma = 4$. The fact that PMAP utilizes only the peaks in the ECG is reflected by the decrease in $\hat{P}_D$ as compared to SMAP. On the other hand, since the peak-picking strategy has a noise-rejecting effect, PMAP produces a slightly lower $\hat{P}_F$. In order to obtain the same $\hat{P}_F$ for both detectors, λ_1 should be chosen somewhat larger in SMAP than in PMAP. For the ROCs in Fig. 3(a)-(b), a suitable operating value for λ_1 would be in the interval 1.1 to 1.3. However, by analyzing the ROC corresponding to class 1 which contains ECGs with high SNRs, a smaller λ_1 is preferably used. On the contrary, if λ_1 is decreased for class 3 and 6, the number of false detections increases rapidly, see Fig. 4.

The worse performance that is yielded for both detectors by using $(K,L) = (3,2)$ or $(4,3)$ is essentially due to an increase in the detection of T-waves. Thus, the choice of γ becomes more critical if these filters are used. However, if class 3 is excluded the difference in performance for the three investigated filters is considerably less. Since tall T-waves usually is a persistent property, these tend to influence the results too much. For $(4,3)$, PMAP performs better than SMAP.

For a decreasing λ_1 one would expect that $\hat{P}_D$ approaches one. However, below a certain value of λ_1 in Fig. 3, $\hat{P}_D$ is almost unchanged. The occurrence of extremely small-amplitude PVCs is the main reason for this behaviour; such PVCs were detected by decreasing λ_1 to 0.4 (not shown in Fig. 3). In addition, a few very wide PVCs were missed due to the error $|\tilde{\theta}_i - \theta_i|$ exceeding 7 samples.

V.3 Discussion

We will now discuss different aspects of the detector and the underlying model assumptions. Several ECG examples are shown which illustrate the treacherous nature of QRS detection. Unless otherwise stated, the signals in Figs. 5-20 are calculated with the parameter values in Table I and ($\lambda_1 = 1.1$, $\gamma = 4$, $(K, L) = (1, 2)$). In these figures, $M(\theta_1)$ and $M^P(\theta_1)$ are plotted by using the statistical parameters estimated in phase II. A detected complex is indicated in $M(\theta_1)$ and $M^P(\theta_1)$ by plotting the value of the threshold $\alpha_1(\theta_1)$ in the interval $[\tilde{\theta}_1 - D, \tilde{\theta}_1 + D]$. If the R-R interval is less than $2D$, the vertical lines in between the detected complexes are omitted. The type-events are marked with horizontal bars.

Abrupt changes in rhythm do not present any problem to the detectors, unless these changes are accompanied by a drastic variation in QRS morphology. Since the rhythm assumptions in the model are very loose, this is not an unexpected result. The eye-closing distance D is the only parameter which imposes a timing restriction in the detectors. The choice of D is limited, since if D is too small a single wide QRS may produce two detections; and even worse, if it is too large one of two closely occurring QRSs may be missed. Paying special regard to the latter aspect, a reasonable choice of this distance is $16 \leq D \leq 20$. Most often when tall P- and T-waves cause false detections, the QRS portion is still detected before these adjacent waves since its magnitude is larger, see Fig. 5 (false detections are indicated by 'F'). In such situations, the number of false detections may be reduced by introducing a differentiated eye-closing, see Sec. VI. If extremely large P- or T-waves occur, it appears impossible to avoid false detections, such as those shown in Fig. 6. The presence of a large spike artefact can preclude the detection of a QRS, if it occurs closer than D from the QRS. This is the case with PMAP in Fig. 7, where in fact the spike is chosen as a type-event. However, with our way of estimating $\hat{\beta}_1$ in (2.23b), sporadically occurring spikes do not affect the threshold in phase II. Since such spikes often have a shape inseparable from certain QRSs, explicit modelling of such artefacts is not suitable for QRS detection purposes.

Linear time-invariant filtering in QRS detection is aggravated because of the conflicting demands of detecting wide PVCs, while rejecting tall P-

and T-waves, and detecting narrow QRS complexes while suppressing high-frequency muscle noise(EMG). In a single ECG recording, the spectral properties of the QRS as well as the noise may vary considerably from time to time. The best performance was obtained for $(K,L) = (1,2)$, which is a BP-filter with center frequency $f_c = 19,6\text{Hz}$. This result agrees with those in [18], where preliminary results indicate that a BP-filter with $f_c = 18\text{Hz}$ produces the highest SNR. If the QRS complex is mismatched to $h_{KL}(k)$ a reduction in SNR is obtained. When $(1,2)$ is used, this reduction is small both for mono- and biphasic QRSs of normal widths, see Fig. 7 on p. 53. By using this filter, the detector becomes rather insensitive to variations in morphology which is illustrated by Fig. 8. Since the spectral variability of the QRS complexes is large, a smaller bandwidth than that of $(K,L) = (1,2)$ seems to be too restrictive as long as a fixed filter is used. However, by letting the filter parameters depend on the noise properties in the ECG, a better performance may be achieved. In Fig. 9, the small amplitude QRS which is corrupted by intermittent muscle noise is possible to distinguish by using a filter with lower f_c , e.g. $(4,3)$.

Slow baseline wandering, which may be due to respiration, rarely affects the performance, see Fig. 10, since it is eliminated by using $K > 0$. However, baseline wandering may sometimes indirectly be the reason for missed complexes, see Fig. 11, where the QRS amplitude depends on the baseline level. Fast baseline shifts which cause false detections, as shown in Fig. 12, are not easily remedied by linear filtering. In fact, such shifts are not really taken care of in the present model, i.e. the assumption that the noise in (2.1) is stationary, zero-mean, Gaussian noise. One way to sort out such false detections, is to make use of the fact that the shifts become essentially monophasic in the bandpass filtered signal, while the QRSs are multiphasic. A very simple algorithm based on this fact has been incorporated in SMAP in Fig. 12. Here it is seen that this algorithm corrects the original four false detections and the two missed complexes.

The necessity of a double-sided a priori probability $p(B_i)$ is well demonstrated in Fig. 13, where a sudden change in polarity occurs. In our model, the probabilities $p(B_i)$ and $\text{Pr}(I_i)$ are considered to be time-invariant. However, for the ECG it is not reasonable to assume that QRS amplitude and

width exhibit stationary properties. Since it is difficult to account for this variation in the signal model, estimates of the statistical parameters may be updated in each observation interval. In SMAP and PMAP, the estimation of statistical parameters is coupled in a simple way to $\tilde{B}_i$ and $\tilde{T}_i$ of the type-events, cf. (2.22)-(2.24). This allows the detector to follow changes in amplitude and width. The ability to follow the former type of changes is controlled by λ_1^T and λ_2^T . Figures 14 and 15 show the output from SMAP for different choices of λ_1^T and λ_2^T when a sudden decrease or increase occurs. In Fig. 15, it is seen that a large λ_2^T yields a fast recovering of the threshold to the current QRS amplitudes. If a moderate sensitivity to amplitude changes is desired, this can also be obtained by including further estimates B_i in the argument in (2.22). In [3] it is claimed that fixed thresholds for amplitude and width are to be preferred, thus implying that these properties are stationary. However, if the episodes in Fig. 14 and 15 should be correctly detected, the fixed values must be chosen very generously, which in noisy situations inevitably results in numerous false detections. An adaptive template-matching algorithm was considered for QRS detection in [19]. That algorithm would probably perform better than SMAP and PMAP in situations of a slow variation in amplitude and width. On the other hand, since these properties sometimes are subject to large changes, (see e.g. Fig. 10, 13, 14 and 15), a larger performance reduction is probably obtained for the template-matching algorithm in such cases.

In some rhythms, e.g. bi- or trigeminy, large-amplitude aberrant beats intermingled with the normal ones may cause problem. If these beats occur within a distance less than N^T from each other, this may obscure the detection of normal ones, see Fig. 16. By decreasing N^T to 200 samples, the normal QRSs are correctly detected. If the aberrant beats are chosen as type-events, the estimates $\hat{\tau}_1$ and $\hat{\tau}_2$ become too large. Apart from the time pattern being different, a similar situation is shown in Fig. 17 in which QRS-like artefacts are intermingled with the normal QRSs. Even if some of these artefacts are excluded by the eye-closing, it is a difficult task to reject all artefacts in Fig. 17, while detecting the normal QRSs in Fig. 16.

Adaptive QRS detection: performance

The only substantial difference in performance of SMAP and PMAP observed on the material was in the detection of very wide QRSs, see Fig. 18 and 19. Since $y(\theta_i)$ and $y(\theta_i - T_i)$ in (2.13) are chosen independently of the peaks such that $x(\theta_i, T_i)$ is minimum, SMAP is more apt to find a wide complex. This fact is also the reason why PMAP performs better than SMAP when $(K, L) = (4, 3)$; SMAP produces more false detections due to T-waves.

VI. DETECTOR IMPROVEMENTS

A number of detector improvements will now be considered, introduced either by changes in the ECG model or by using additional techniques.

Differentiated eye-closing. The P- and T-waves most often occur within a certain distance from the QRS (except for e.g. P-waves in complete A-V block). A means for reducing the number of false detections due to T-waves is to apply a differentiated eye-closing after each detected QRS [19], [20]. For example, in [20] this was accomplished by a refractory period of 200ms which is followed by a linearly decreasing threshold. Furthermore, a constant threshold was employed before each QRS for P-wave rejection. By introducing such an eye-closing in our detector, the magnitude of $\alpha_i(\theta_i)$ will depend on the distance from previously detected complexes $\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_{i-1}$. Thus, the differentiated eye-closing will apply equally to a segment before as well as after each detected QRS. This property is incorporated into the model by considering the arrival times as if they occur according to an ordinary renewal process, i.e. the intervals $(\theta_i - \theta_{i-1})$ are independent and identically distributed. The joint arrival time density will be completely characterized if the so-called hazard function is known [21]. For the model in this paper, the hazard function which yields $\Pr(\underline{\theta})$ is defined by

$$h_1(k) = \begin{cases} 0 & 0 \leq k \leq D-1 \\ \mu & D \leq k \end{cases} \quad (6.1)$$

where μ is a constant. The function in (6.1) implies that there is zero probability for arrival times occurring closer to each other than D . In order to obtain the desired eye-closing property, the constant μ is replaced by e.g. an increasing function in the interval $[D, D_1]$ followed by a constant for $k > D_1$.

Noise-dependent threshold. Since $\Pr(q)$ is assumed to be uniform (see Sec. II.5), the threshold $\alpha_i(\theta_i)$ does not depend on the noise level N_0 . An improved performance is most likely obtained if $\alpha_i(\theta_i)$ is adapted in relation to the prevailing noise level. A noise-dependent threshold will reduce the difference in performance between e.g. class 1 and 6 obtained for the same value of λ_1 , see Fig. 4(a).

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Such a dependency could formally be introduced by choosing $\text{Pr}(q)$ such that $\ln(p_{i-1}/p_i)$ in (2.14) is constant, which is

$$p_i - a p_{i-1} = 0, \quad i = 1, \dots, n \quad (6.2)$$

In that case, the resulting threshold does depend on N_0 , but is independent of the number of picked maxima. Subject to the constraint that a probability mass distribution sums up to one, the a priori probability which fulfils (6.2) is given by

$$\text{Pr}(q) = \begin{cases} a^q \frac{1-a}{1-a^{n+1}} & q = 0, \dots, n \\ 0 & \text{otherwise} \end{cases} \quad (6.3)$$

where $0 < a < 1$. By inserting (6.3) in (2.14), one obtains

$$\alpha_i(\theta_i) = \frac{\beta_1}{2} - \frac{N_0}{4E_V\beta_1} \ln a - \ln \chi(\tilde{\theta}_1, \dots, \theta_i). \quad (6.4)$$

The parameter a thus controls the dependency of the $\text{SNR} \sim E_V/N_0$ on the threshold.

In order to employ the threshold (6.4), an estimate of N_0 must be determined. This problem will not be considered here.

"Rhythm-conditioned" detection. The detector treats the observation interval in an equal manner, whether the previously estimated RR-intervals are of almost constant length or exhibit a large variation. By introducing information about the properties of preceding RR-intervals in the detector, errors of the type seen in Figs. 11, 16 and 20 could be avoided. We will now outline a simple extension of the detectors in which such information is included in Phase I. This is done in the following way:

Adaptive QRS detection: performance

1. If the preceding RR-intervals exhibit a small variation, the detector first decides if a QRS (type-event) has occurred or not at the predicted, regular time instance. If no type-event is detected, the detector operates as previously described, cf. Sec. II.
2. If the variation of preceding RR-intervals is not negligible, the detector disregards the rhythm history, and instead analyzes the observation interval as if the QRS complexes occur uniformly.

No rhythm information is introduced in the second phase, since regularly occurring QRS complexes, with sufficient magnitude in $M(\Theta_1)$, are already found in Phase I.

Certain rhythms are manifested by that every second or third beat in a regular pattern is replaced by a premature beat (bigeminy and trigeminy, respectively). If the lengths of the intervals which surround the premature beat sum up to approximately two normal RR-intervals, these rhythms are here considered as being regular.

A prerequisite for "rhythm-conditioned" QRS detection is an algorithm which supplies the detector with a decision about the regularity of the preceding RR-intervals, and an estimate of the "average" RR-interval if the rhythm is judged regular. A related type of problem is the classification of persistent and transient rhythms. An elegant approach to that problem has been presented in [23], [24], where classification schemes are derived from simple, dynamic models describing various types of rhythms. For our purposes, the following, extremely simple, scalar model for a regular rhythm is considered

$$x_k = x_{k-1} \quad (6.5)$$

$$r_k = x_k + w_k. \quad (6.6)$$

The natural, small variation in the observed RR-intervals r_k is modelled by a zero-mean, white, Gaussian sequence w_k added to the constant length interval x_k . The initial condition x_0 is assumed to be a Gaussian random variable with mean $\bar{x}_0$ and variance P_0 . The variance of w_k is denoted

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with R . The minimum-variance estimate of x_k based on $r_1, \dots, r_{k-1}$ is given by the filter [25]

$$\hat{x}_k = \hat{x}_{k-1} + K_k \tilde{r}_k \quad (6.7)$$

$$\tilde{r}_k = r_k - \hat{x}_{k-1} \quad (6.8)$$

which is initialized with $\hat{x}_0 = \bar{x}_0$. The gain K_k in (6.7) is defined by the recursion

$$V_k = \Sigma_{k-1} + R \quad (6.9)$$

$$K_k = \Sigma_{k-1} / V_k \quad (6.10)$$

$$\Sigma_k = (1 - K_k) \Sigma_{k-1} \quad (6.11)$$

which is initialized with $\Sigma_0 = P_0$. The prediction errors $\tilde{r}_1, \dots, \tilde{r}_k$ form a zero-mean, white sequence with variance V_k .

A standard procedure for detecting anomalies in the observed sequence is to check the magnitude of the quantity $\tilde{r}_k^2 / V_k$. If this quantity exceeds a certain threshold, an anomaly is detected. In order to avoid declaring e.g. bigeminy as a rhythm with large variation, the following test procedure and filter updating has been applied.

- a. A regular rhythm is observed if at least n_c consecutive prediction errors are such that $|\tilde{r}_k'|$ does not exceed the threshold δ ($\tilde{r}_k' = \tilde{r}_k / \sqrt{V_k}$). The filter is updated by (6.7).
- b. An irregular rhythm is detected if $\tilde{r}_k' > \delta$. The filter is reinitialized by setting $\bar{x}_0$ equal to the average of the two most recent RR-intervals.
- c. If $\tilde{r}_k' < \delta$, this could be an indication of a rhythm in which two adjacent intervals sums up to two normal RR-intervals. Therefore, the filter is updated by $\hat{x}_k = \hat{x}_{k-1}$ and a decision is deferred until the next observation. Using the next observation, the rhythm is considered regular if

$$\frac{|(r_{k-1}+r_k)/2 - \hat{x}_{k-1}|}{\sqrt{V_k}} < \delta,$$

and the filter is updated by (6.7), but with r_k replaced by $(r_{k-1}+r_k)/2$. If this condition is not fulfilled, the filter is reinitialized as in b.

Two examples with different rhythms are shown in Figs. 21 and 22, together with an estimate of x_k in the intervals with regular rhythm. A lower bound was put on the filter gain K_k in order to retain a certain sensitivity of the filter (0.1). The following parameter values were used: $n_c = 5$, $\delta = 0.8$, $P_0 = 25$ and $R = 64$.

Having detected a regular rhythm, the following is done in the detector. If a normal RR-interval precedes the observation interval, the detector searches the interval $[\hat{x}_k - \delta\sqrt{V_k}, \hat{x}_k + \delta\sqrt{V_k}]$ for a QRS complex. If nothing is detected in that interval, the interval $[2\hat{x}_k - \delta\sqrt{V_k}, 2\hat{x}_k + \delta\sqrt{V_k}]$ is then searched. If the preceding interval is such that $\tilde{r}_k < -\delta_k$, the interval $[2\hat{x}_k - r_k - \delta\sqrt{V_k}, 2\hat{x}_k - r_k + \delta\sqrt{V_k}]$ is only searched.

The above algorithm for rhythm-conditioned detection was implemented with SMAP and PMAP. It was found that the missed QRS complexes in Figs. 11, 16 and 20 now were correctly detected. Furthermore, the number of errors caused by a large-amplitude T-wave that "overshadowed" a QRS complex was reduced.

VII. CONCLUSIONS

The use of a model-based approach to QRS detection has been studied in this part. Two different simplified detectors (SMAP and PMAP) were implemented. The performance of the detectors was compared by means of a material of ECGs including various difficult types of signals. The results indicate that the detectors are able to recognize a wide range of QRS morphologies. Both detectors adapt quickly to new signal situations: a property due to the idea of estimating the statistical parameters from the waveforms which delimit the observation interval. While SMAP in general presents a slightly better performance than PMAP, the improvement is at the expense of an increase in computational complexity associated with SMAP.

APPENDIX

Simplification of non-linear scheme

If the probability for i waveforms is non-increasing and the lower limit β_1 in the a priori probability density $p(B_i)$ is such that

$$\beta_1^2 > \frac{N_0 \max_i \ln \frac{p_{i-1}}{p_i}}{2E_v \cdot \min_{T_i} (1 - \rho(T_i))} \quad (A.1)$$

where $|\rho(T_i)| < 1$ for $\tau_1 \leq T_i \leq \tau_2$ and $i = 1, \dots, n$, then identical estimates $\tilde{q}$ and $\tilde{\theta}$ are obtained whether $F(x(\theta_i, T_i))$ is used in (2.10) or replaced by

$$F'(x(\theta_i, T_i)) = |x(\theta_i, T_i)| \quad (A.2)$$

This fact is shown by first rewriting $F(x(\theta_i, T_i))$ on p. 38 as

$$F(x(\theta_i, T_i)) = |x(\theta_i, T_i)| + g(x(\theta_i, T_i)) \quad (A.3)$$

where

$$g(x(\theta_i, T_i)) = \begin{cases} 0 & |x| < \beta_1 \\ \frac{(|x| - \beta_1)^2}{2\beta_1} & \beta_1 \leq |x| \leq \beta_2 \\ \frac{(|x| - \beta_1)^2 - (|x| - \beta_2)^2}{2\beta_1} & \beta_2 < |x| \end{cases} \quad (A.4)$$

and $x = x(\theta_i, T_i)$. Clearly, $g(x(\theta_i, T_i))$ is a non-negative function. Performing the maximization with respect to T_i , an estimate of the arrival time θ_i is obtained as that value which maximizes Δ_i in (2.13); formally

$$\begin{aligned} \tilde{\theta}_i = \arg \max_{\theta_i} & \left[(1 - \rho(\tilde{T}_i)) \left(|x(\theta_i, \tilde{T}_i)| + g(x(\theta_i, \tilde{T}_i)) \right) + \right. \\ & \left. + \rho(\tilde{T}_i) \cdot \frac{\beta_1}{2} - \alpha_i(\theta_i) \right]. \end{aligned} \quad (A.5)$$

Since $p_{i-1} \geq p_i$, arrival time estimates are determined only as long as Δ_i remains positive. Now, if

$$\alpha_i(\theta_i) < \left(1 - \rho(\tilde{T}_i)\right) \beta_1 + \rho(\tilde{T}_i) \frac{\beta_1}{2},$$

the argument in (A.5) will be positive whether $g(x(\theta_i, T_i))$ is included or not. By using (2.14) and rearranging the above inequality, one obtains

$$\beta_1^2 > \frac{N_0 \ln \frac{p_{i-1}}{p_i}}{2E_v \left(1 - \rho(\tilde{T}_i)\right)}.$$

Since $\tilde{T}_i$ is not known, one simply choose that T_i which maximizes the righthand side. By choosing p_{i-1} and p_i in a similar way, one obtains (A.1).

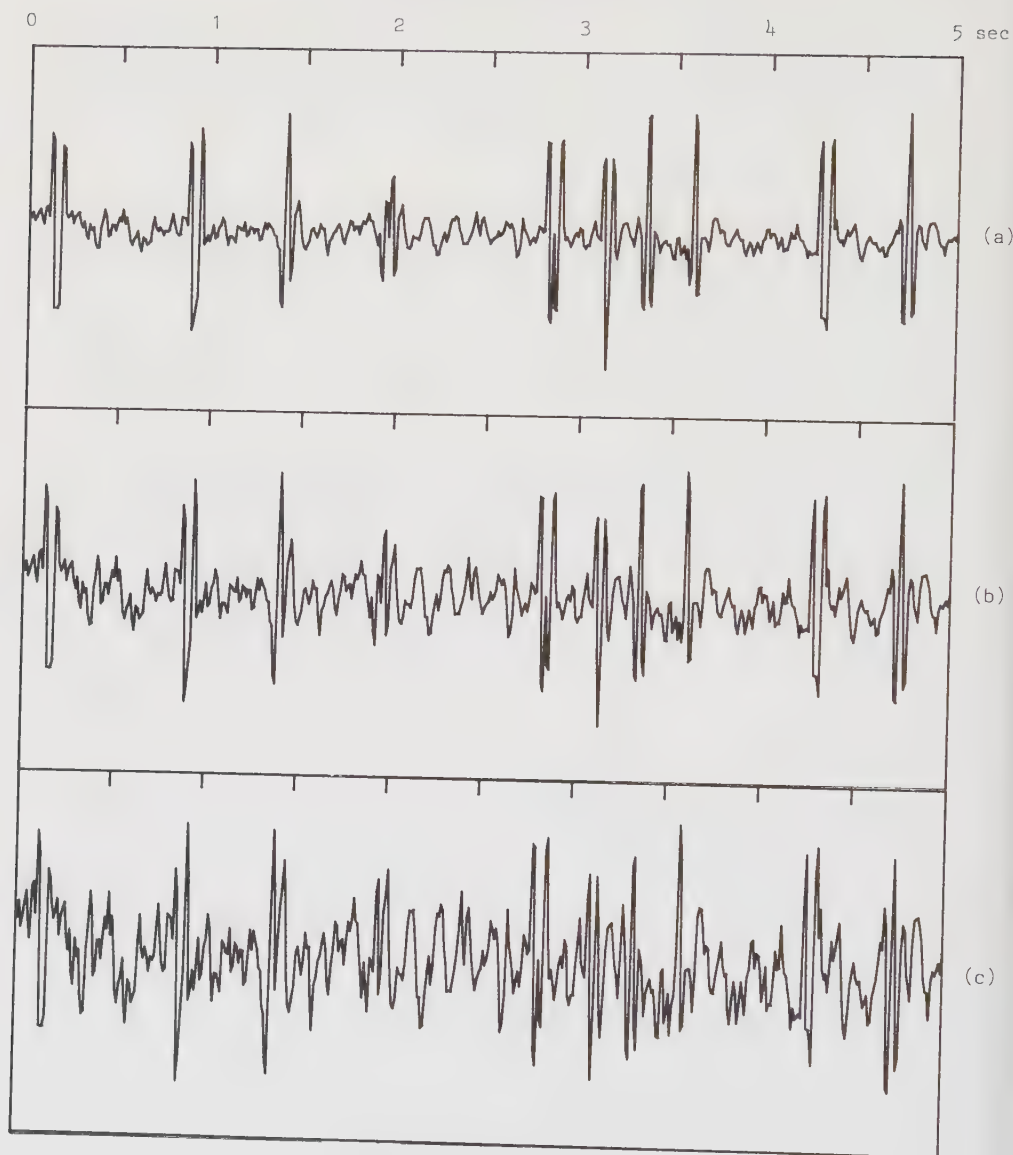


Fig. 1. Example of ECG as generated by the model for three different SNRs. (a) 30dB, (b) 25dB and (c) 20dB.

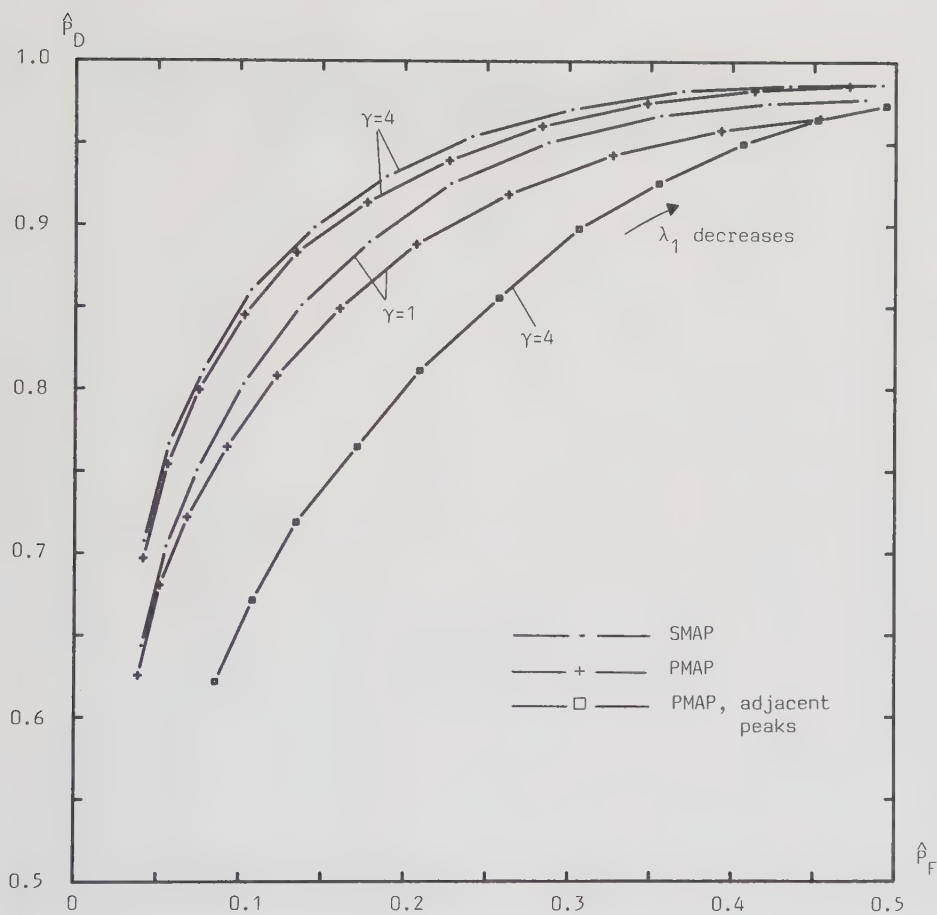


Fig. 2(a). ROCs obtained for simulated ECGs. SNR is equal to 20dB.

λ_1 is decreased from $\lambda_{1,\max} = 1.6$ in steps of $\Delta\lambda_1 = 0.1$.

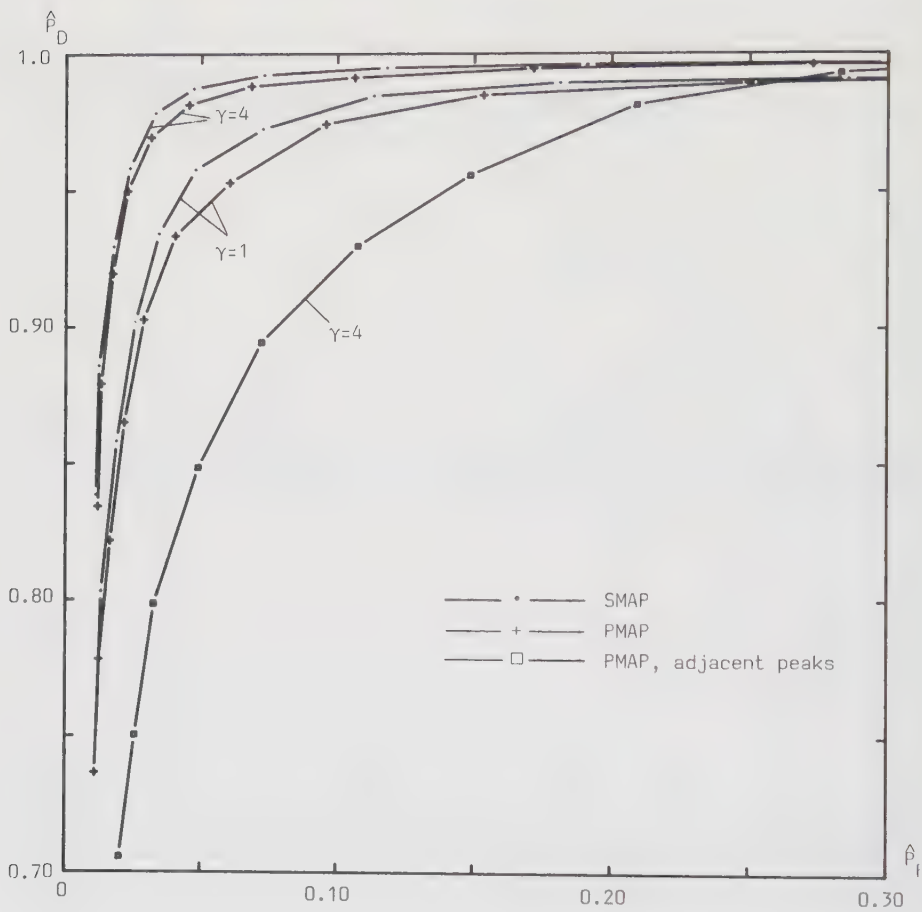


Fig. 2(b). ROCs obtained for simulated ECGs. SNR = 25dB, $\lambda_{1,\max} = 1, 2$ and $\Delta\lambda_1 = 0.1$.

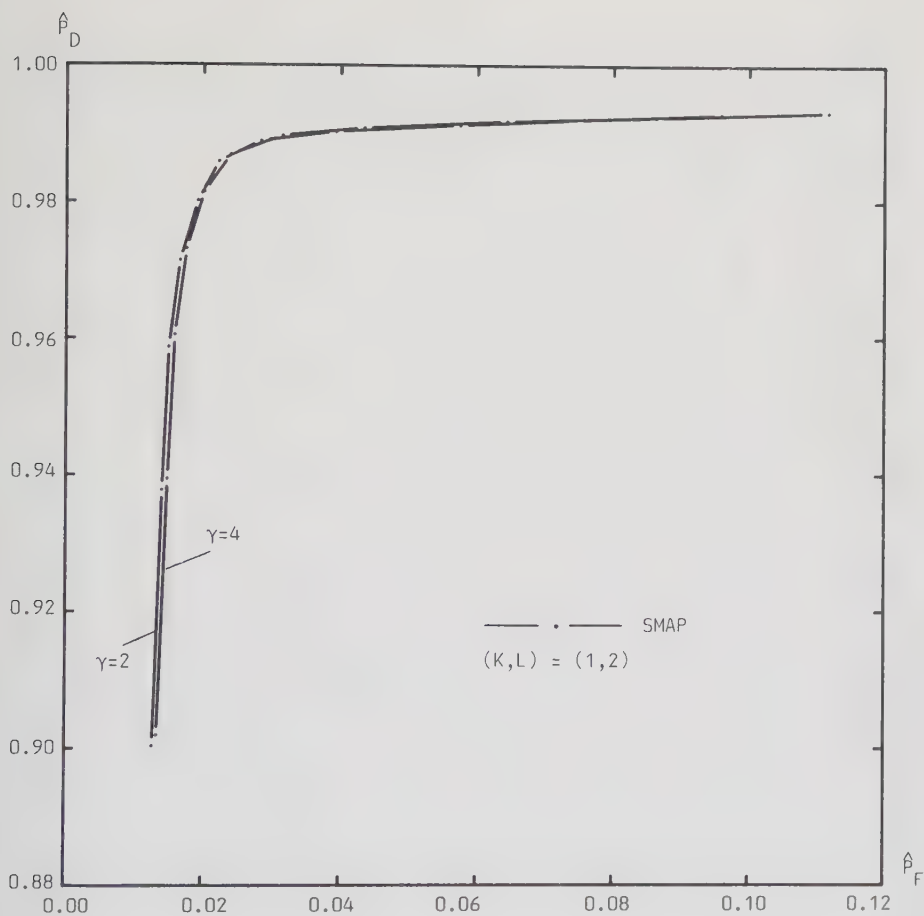


Fig. 3(a). ROCs obtained for the total ECG material.

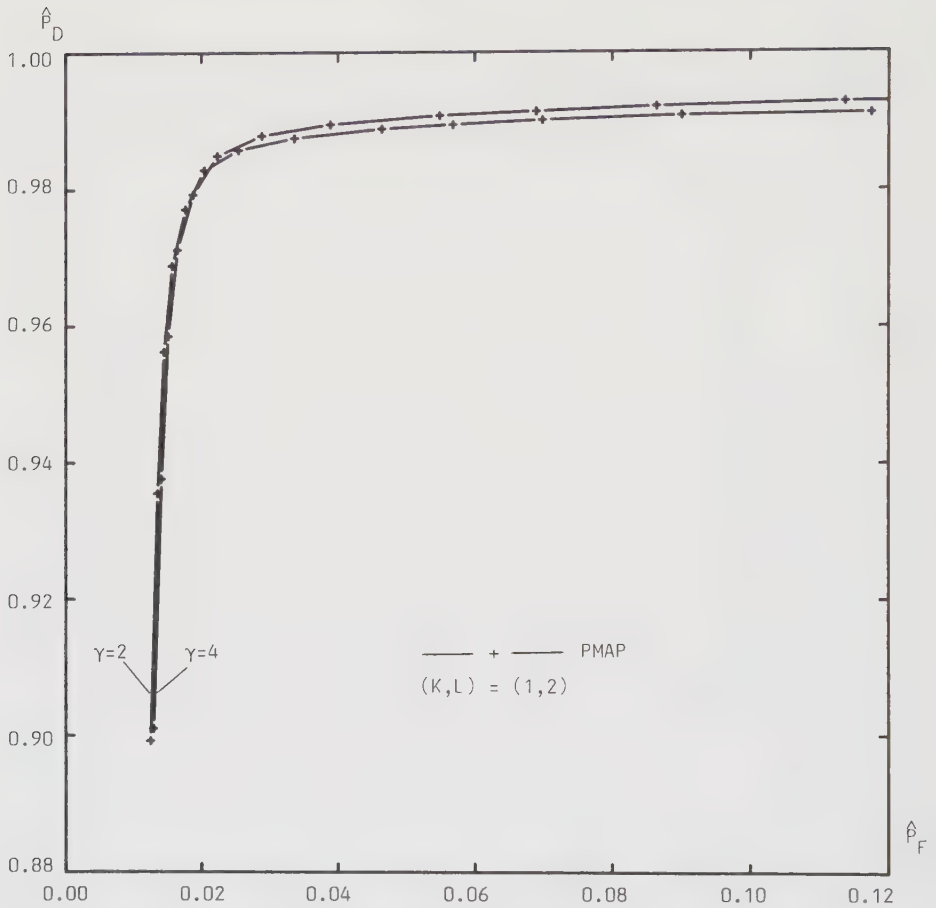


Fig. 3(b). ROCs obtained for the total ECG material.

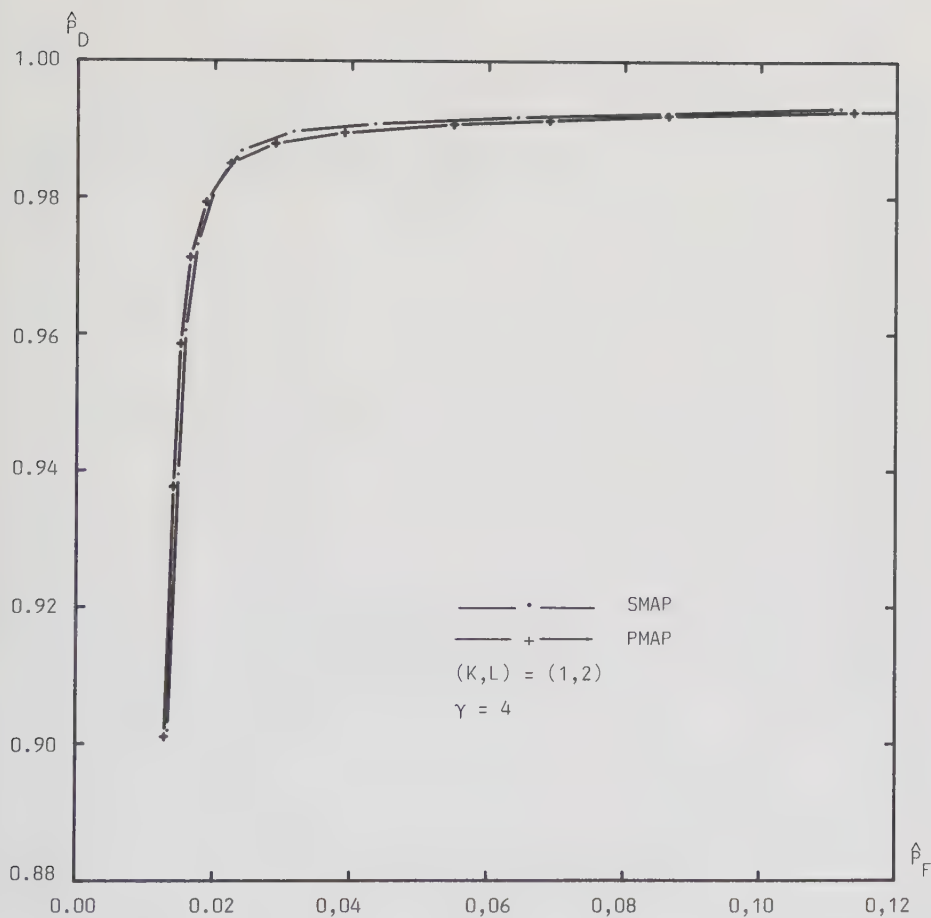


Fig. 3(c). Comparison of ROCs obtained for the total ECG material.

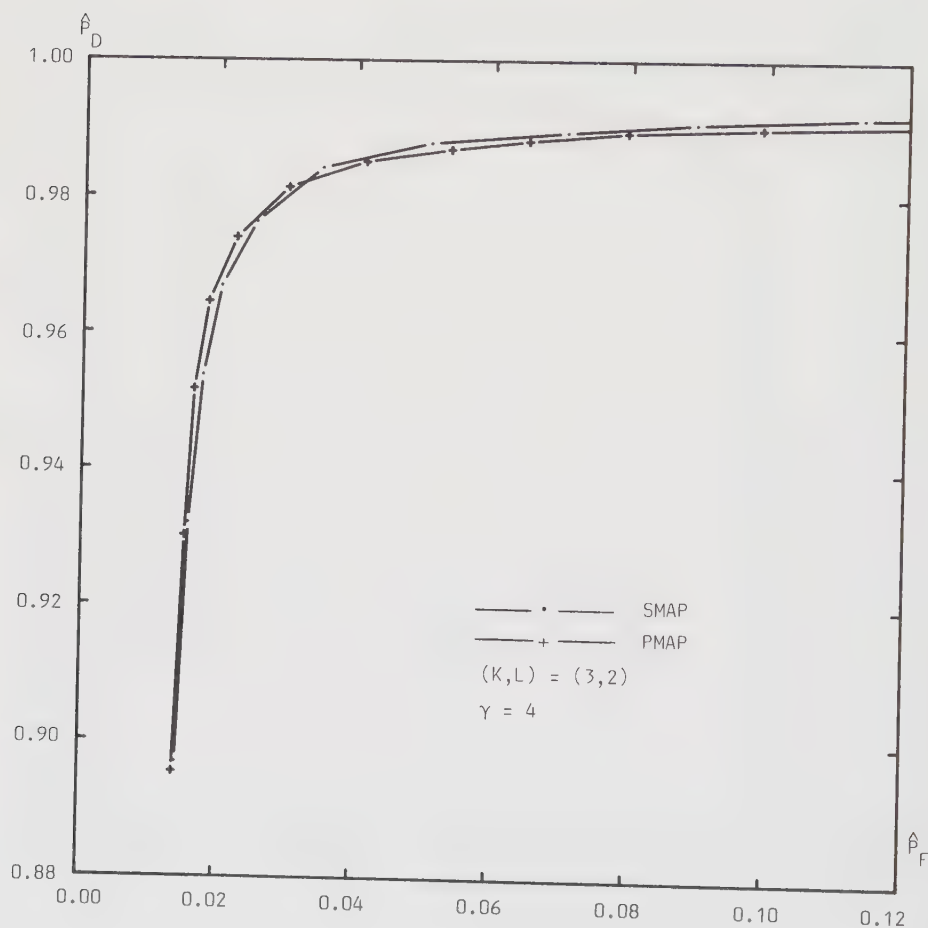


Fig. 3(d). ROCs obtained for the total ECG material.

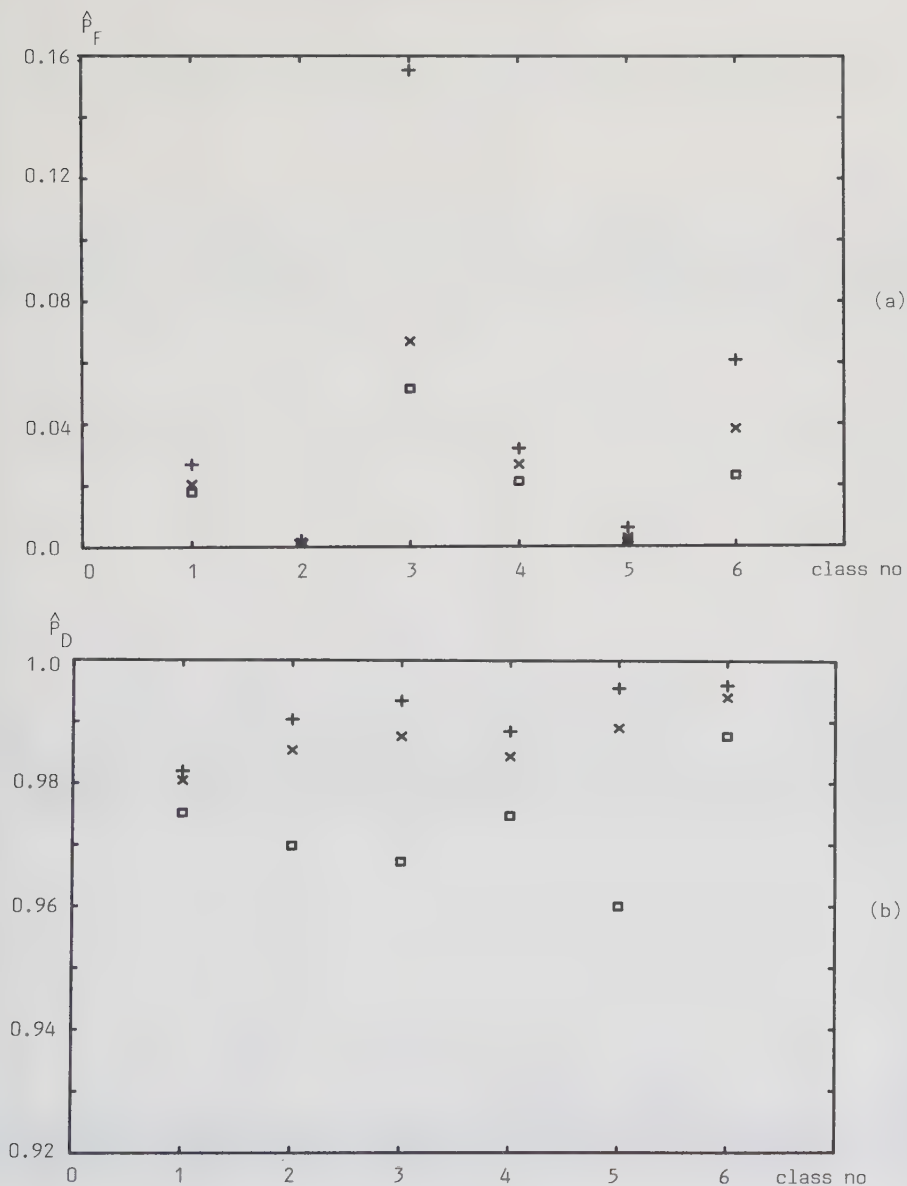


Fig. 4(a). False detection rate and
 (b). true detection rate for SMAP as obtained for each class.
 + $\lambda_1=1.0$, x $\lambda_1=1.2$, □ $\lambda_1=1.4$, $\gamma=4$, $(K,L) = (1,2)$

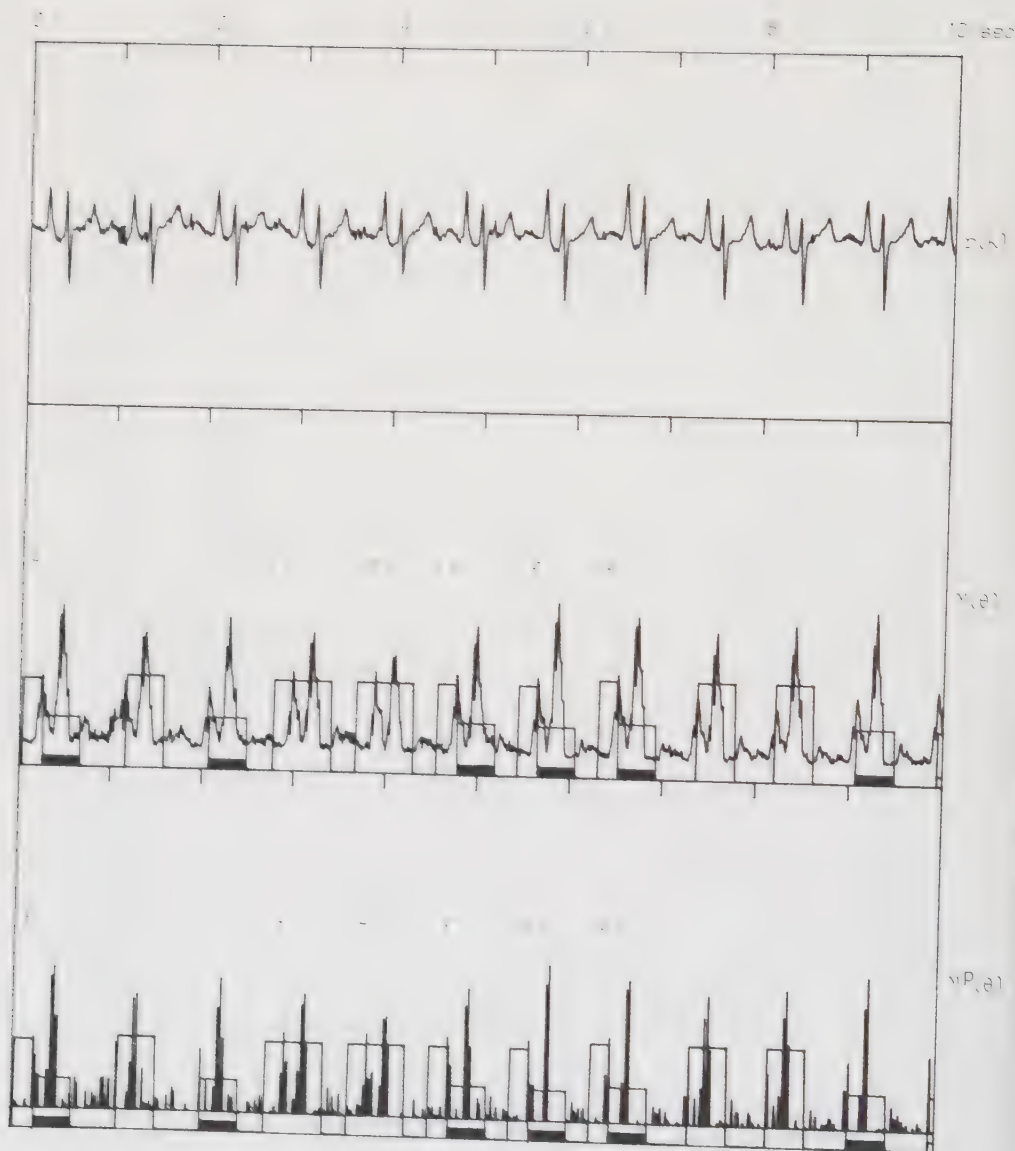


Fig. 5. Large-amplitude P-waves which cause 6 false alarms. For explanation of the thresholds plotted in $M(\theta)$ and $M^P(\theta)$, see the text.

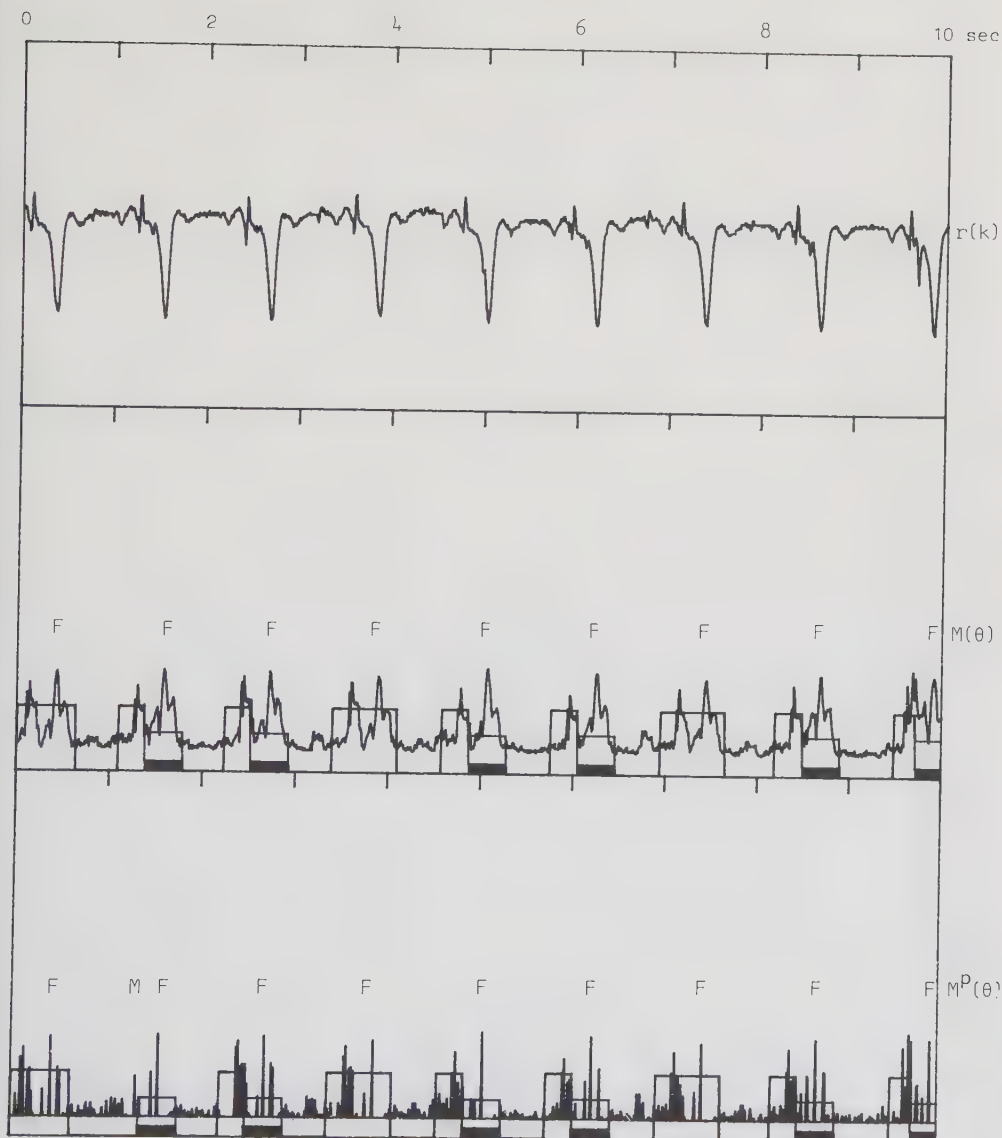


Fig. 6. Extremely large T-waves of which some are detected as type-events.

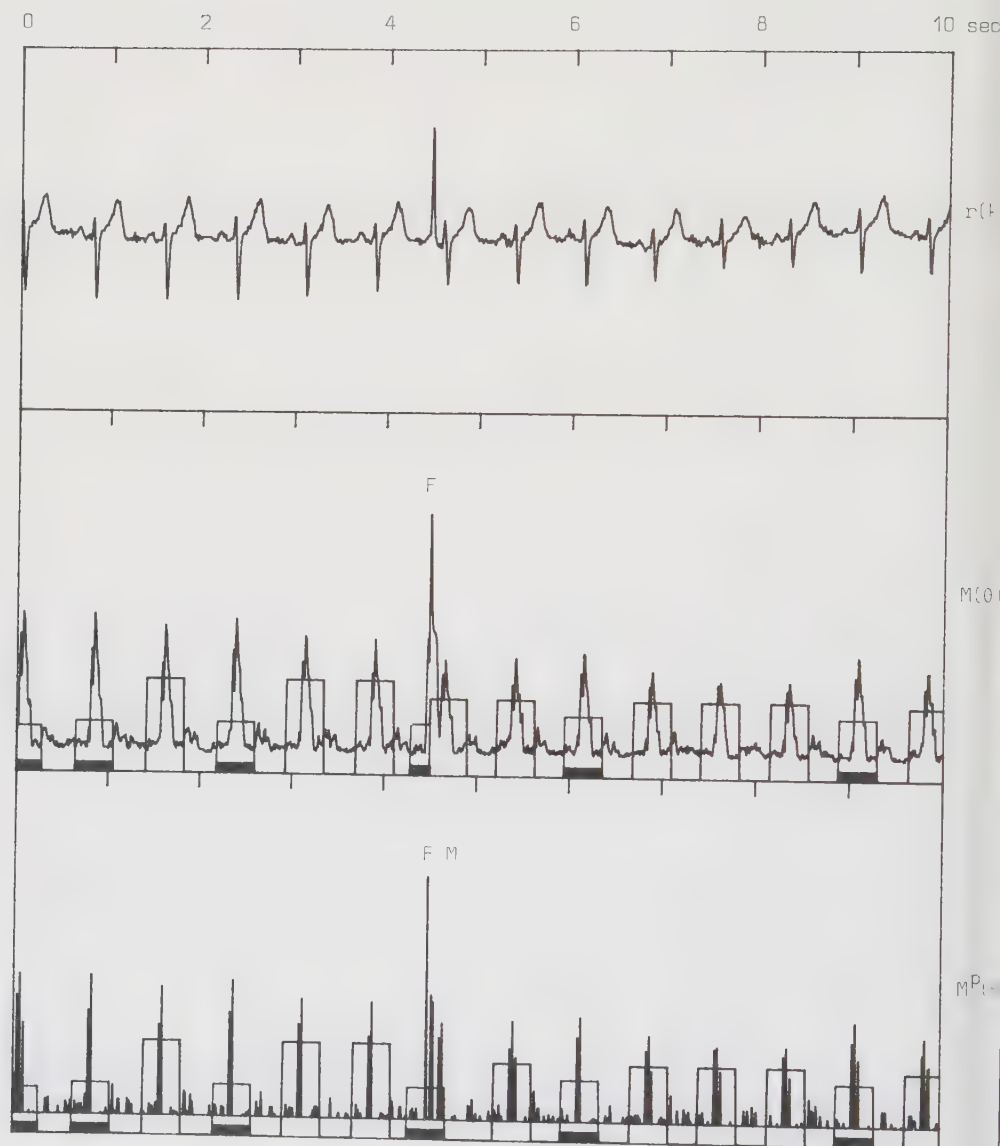


Fig. 7. Normal sinus rhythm interrupted by a spike artefact.

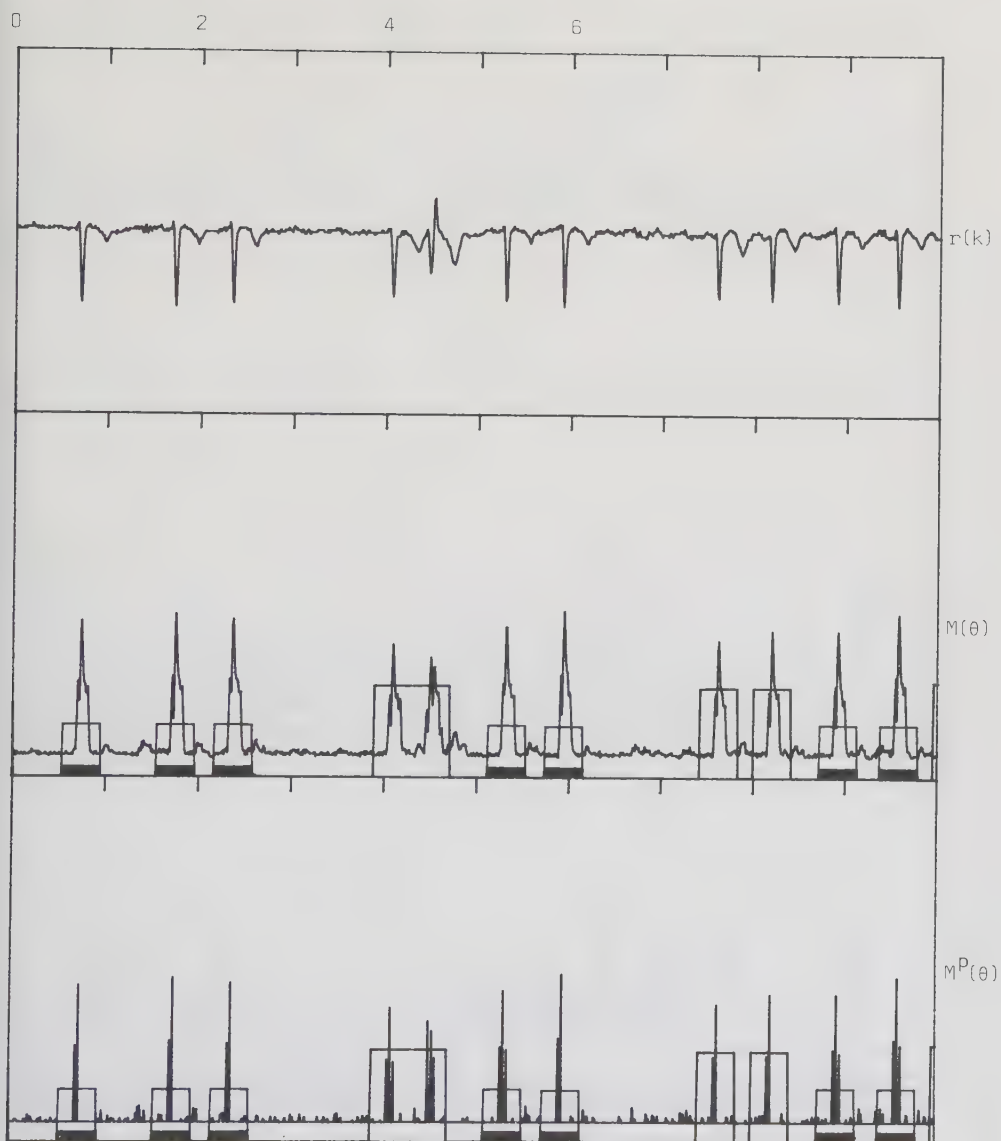


Fig. 8. Sudden change in QRS morphology.

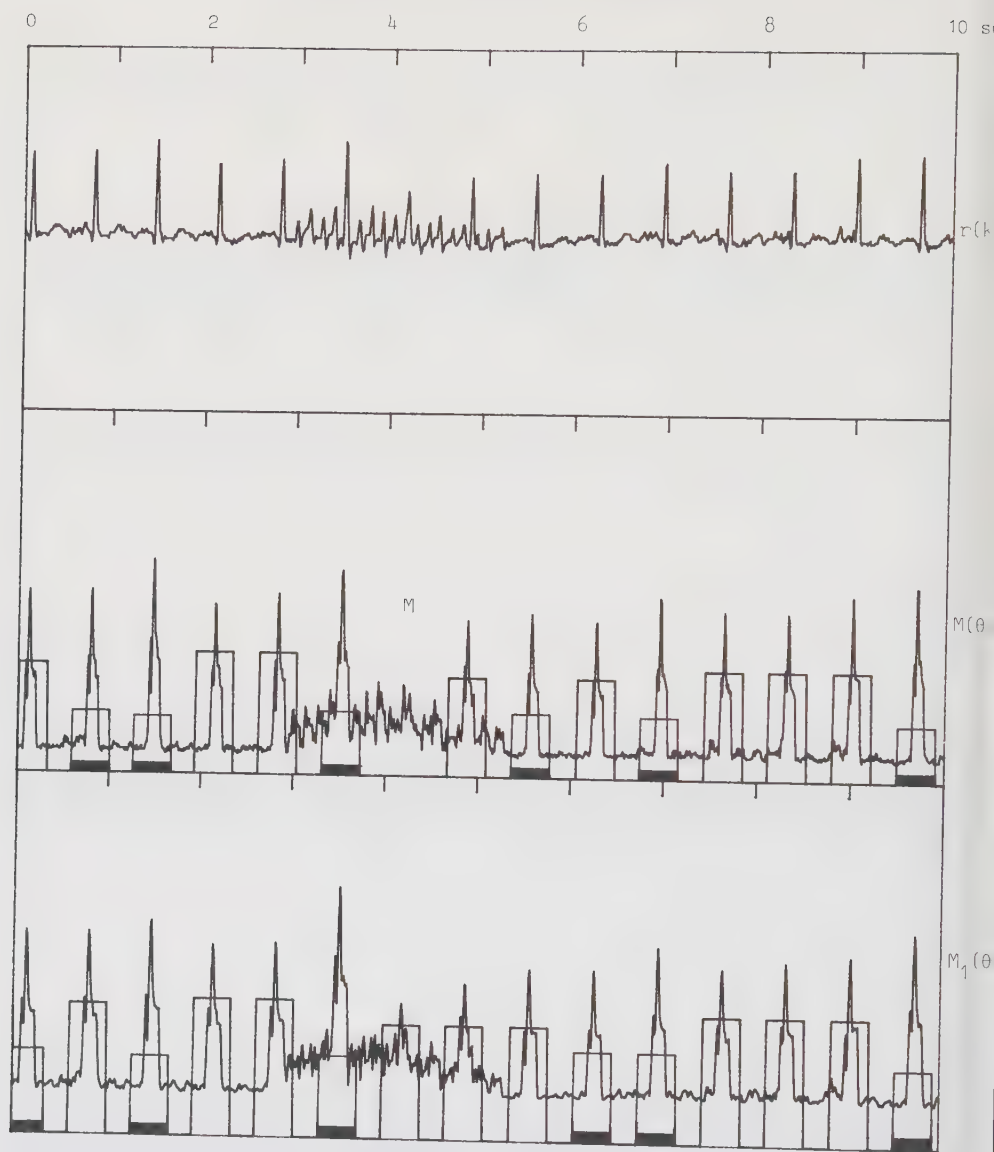


Fig. 9. Small-amplitude QRS complex corrupted by intermittent muscle noise. The signal $M_1(\theta)$ is calculated with the filter (4,3).

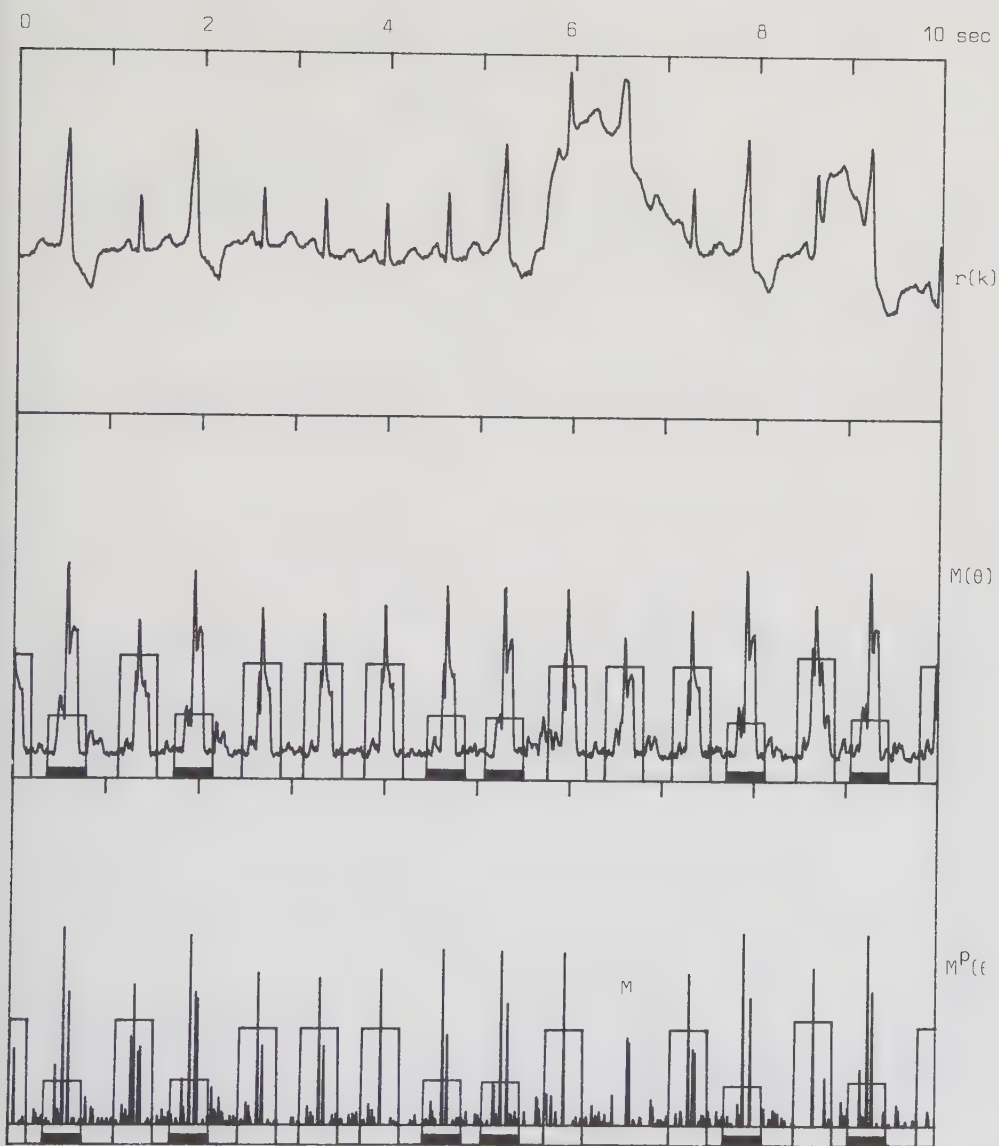


Fig. 10. Slow baseline wandering.

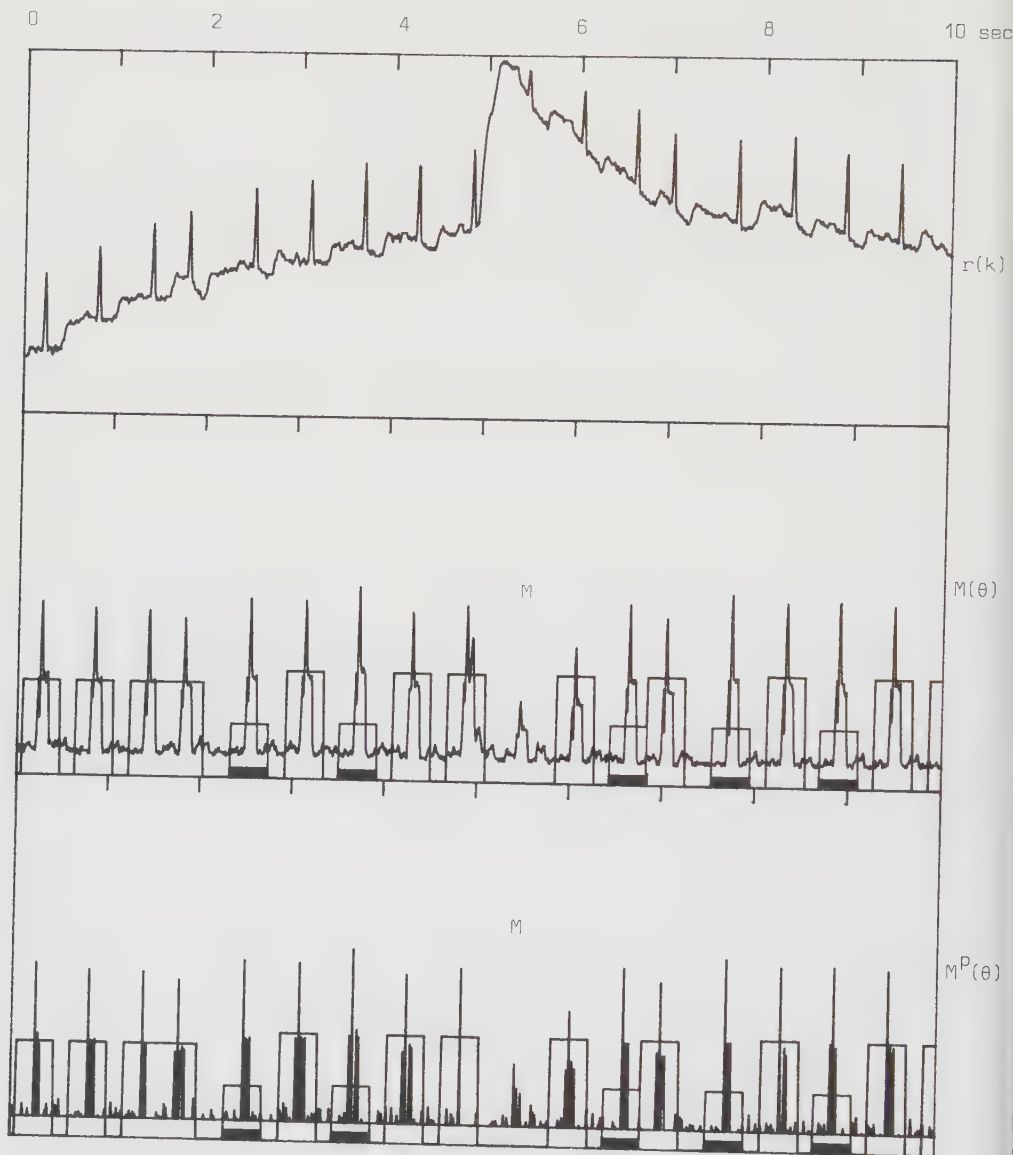


Fig. 11. Slow baseline wandering causing a decrease in QRS amplitude.

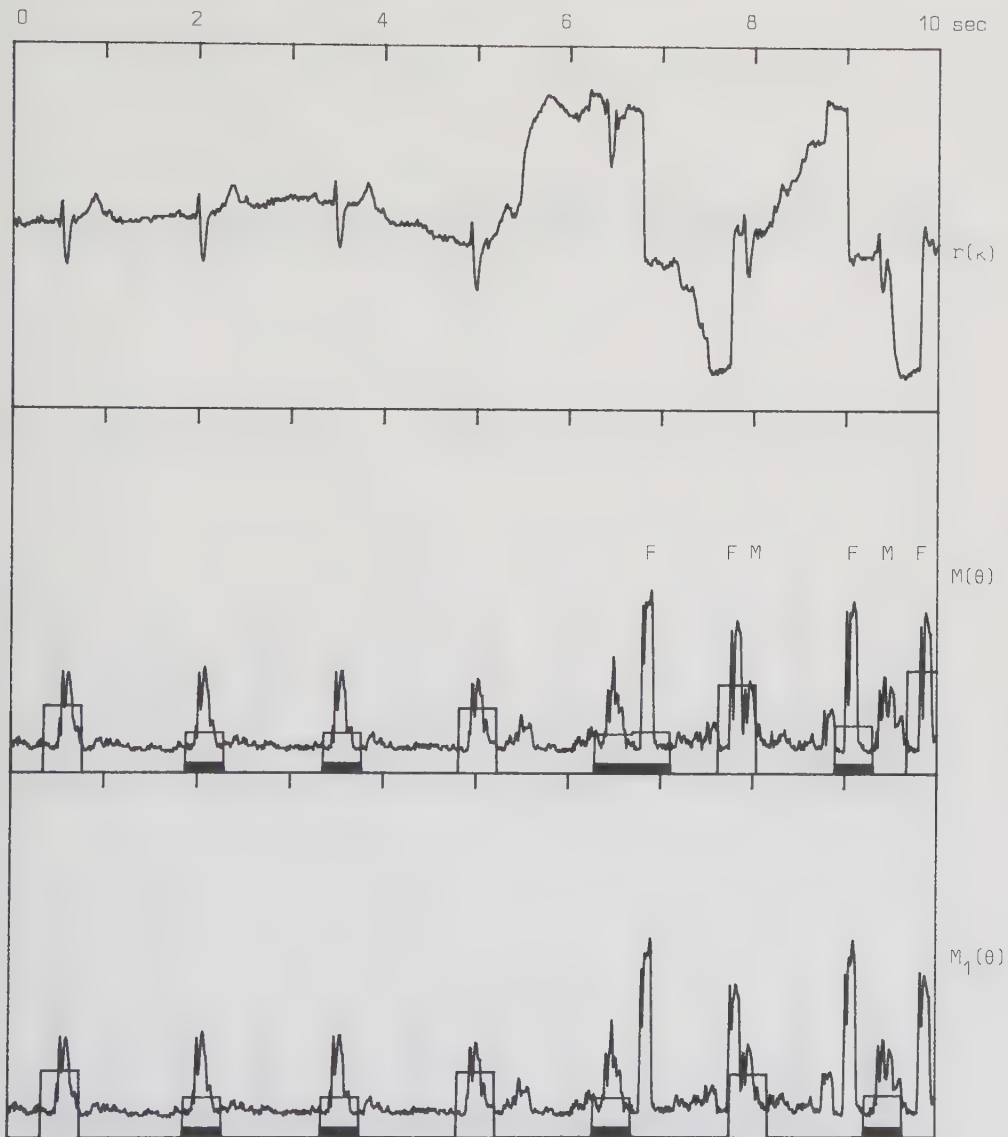


Fig. 12. Baseline shifts which causes 4 false alarms and two missed complexes. The decisions in $M_1(\theta)$ are based on a simple algorithm for sorting out shifts.

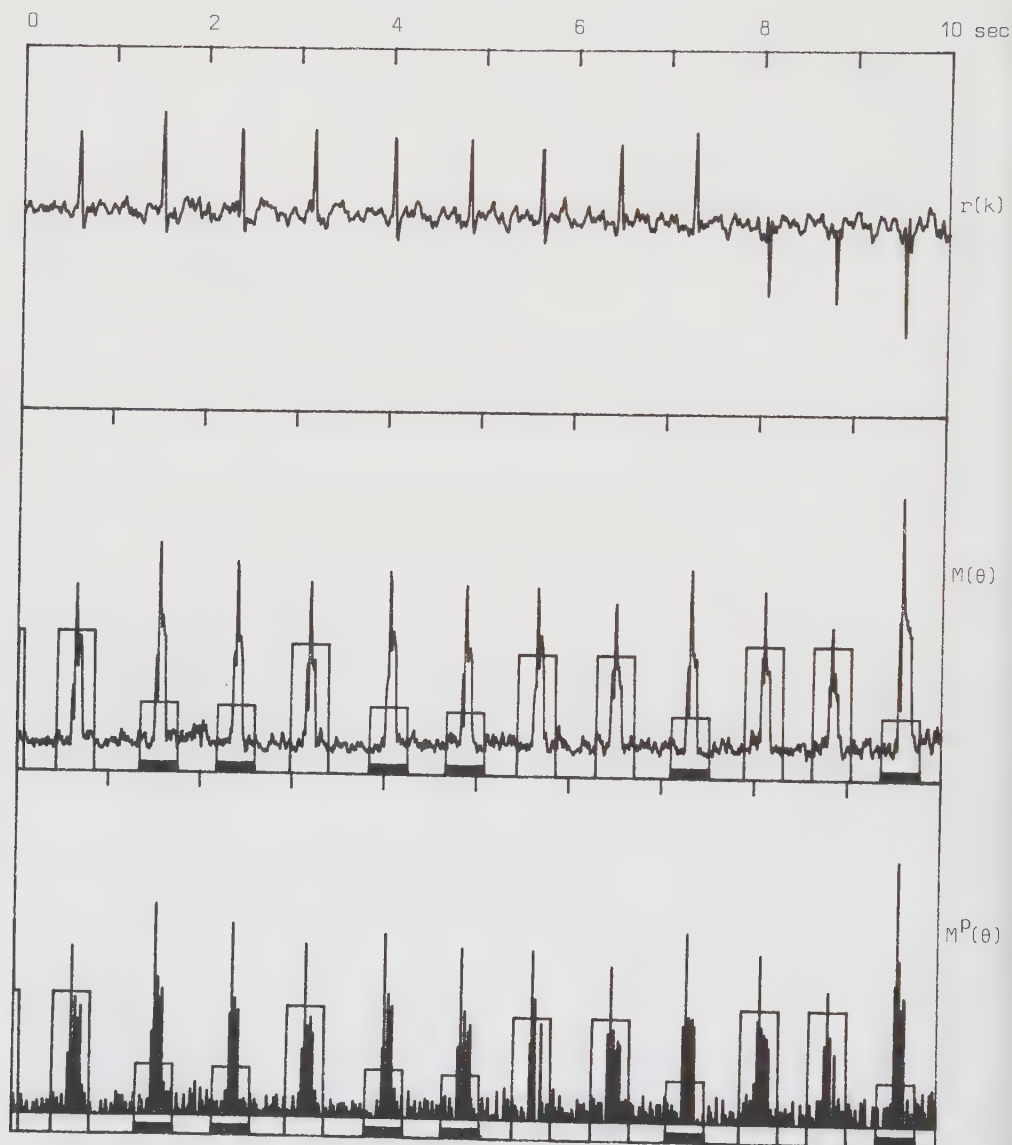


Fig. 13. Normal sinus rhythm with sudden change in polarity.

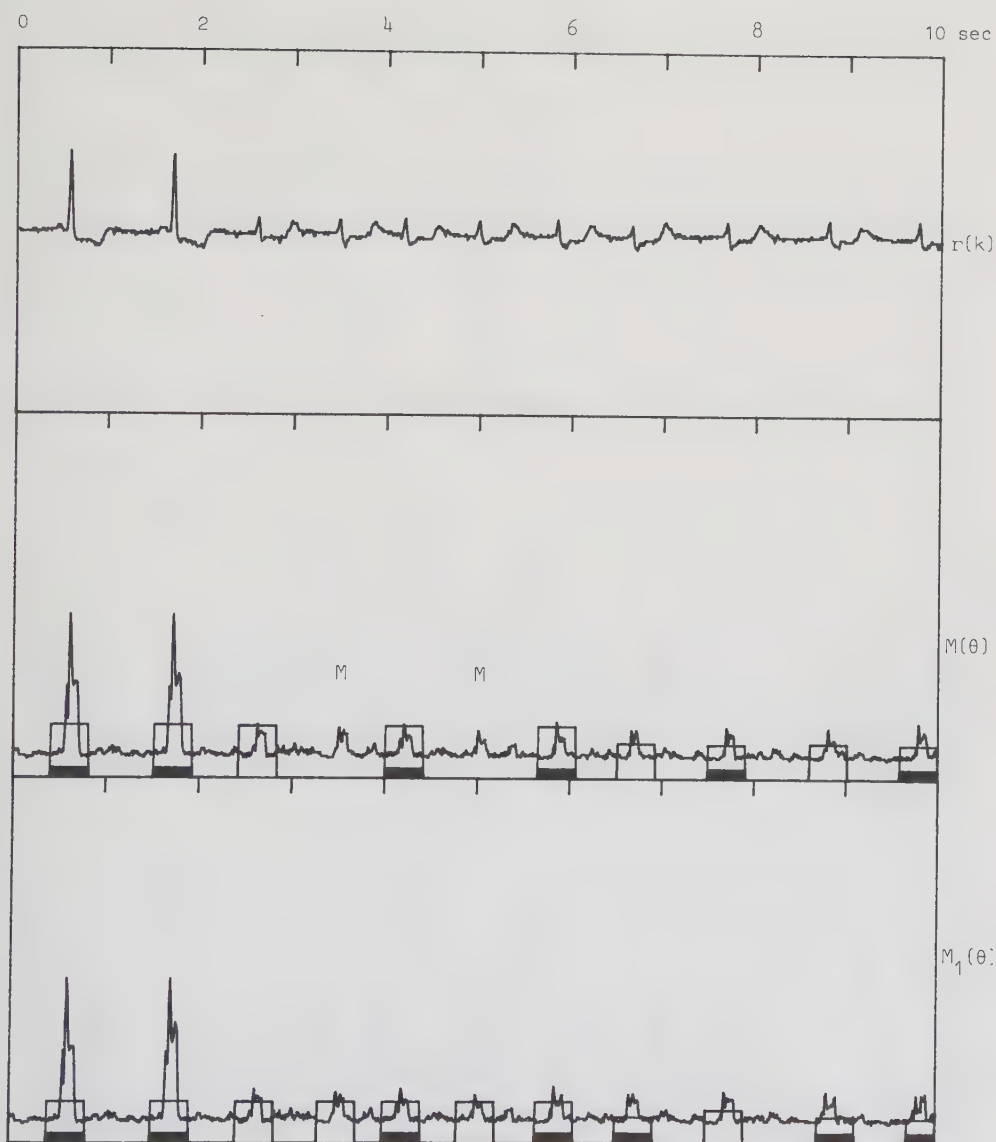


Fig. 14. Drastic decrease in QRS amplitude. The signals $M_1(\theta)$ is calculated with $(\lambda_1^T, \lambda_2^T)$ equal to $(0.15, 2.0)$.

Adaptive QRS detection: performance

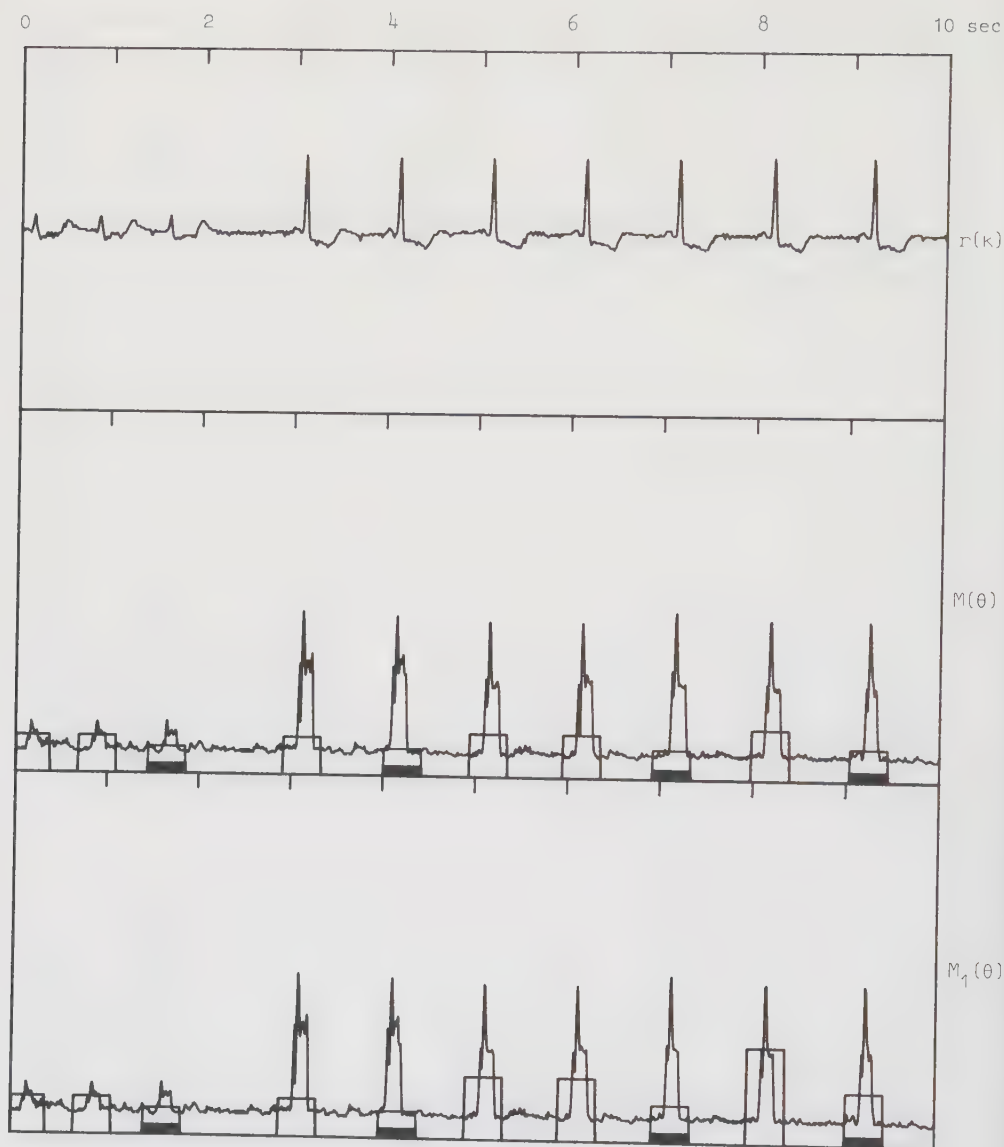


Fig. 15. Drastic increase in QRS amplitude. The signals $M_1(\theta)$ is calculated with $(\lambda_1^T, \lambda_2^T)$ equal to $(0.15, 3.0)$.

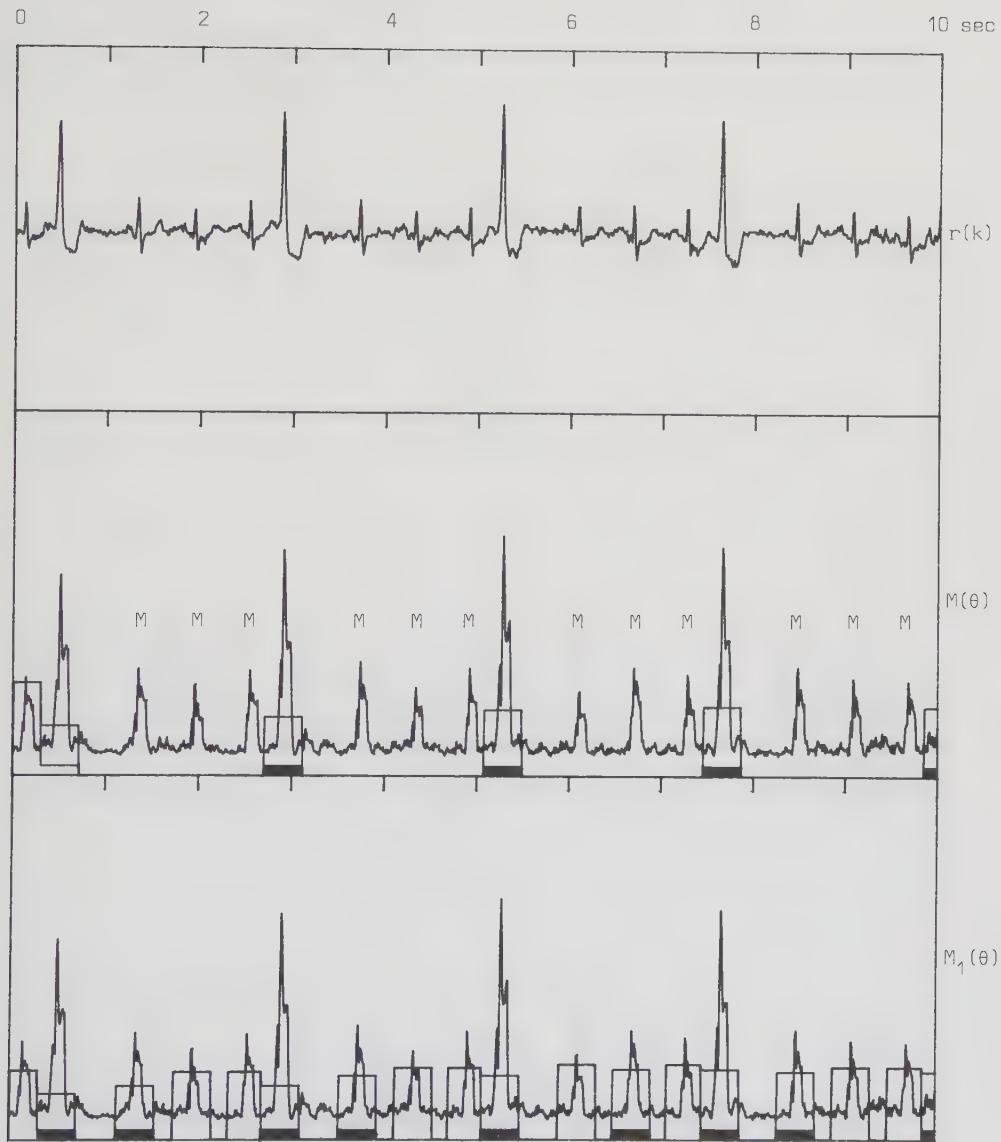


Fig. 16. Normal QRS complexes intermingled with large-amplitude, aberrant ones. The signal $M_1(\theta)$ is calculated with $N^T = 200$ samples (i.e. 2 sec).

Adaptive QRS detection: performance

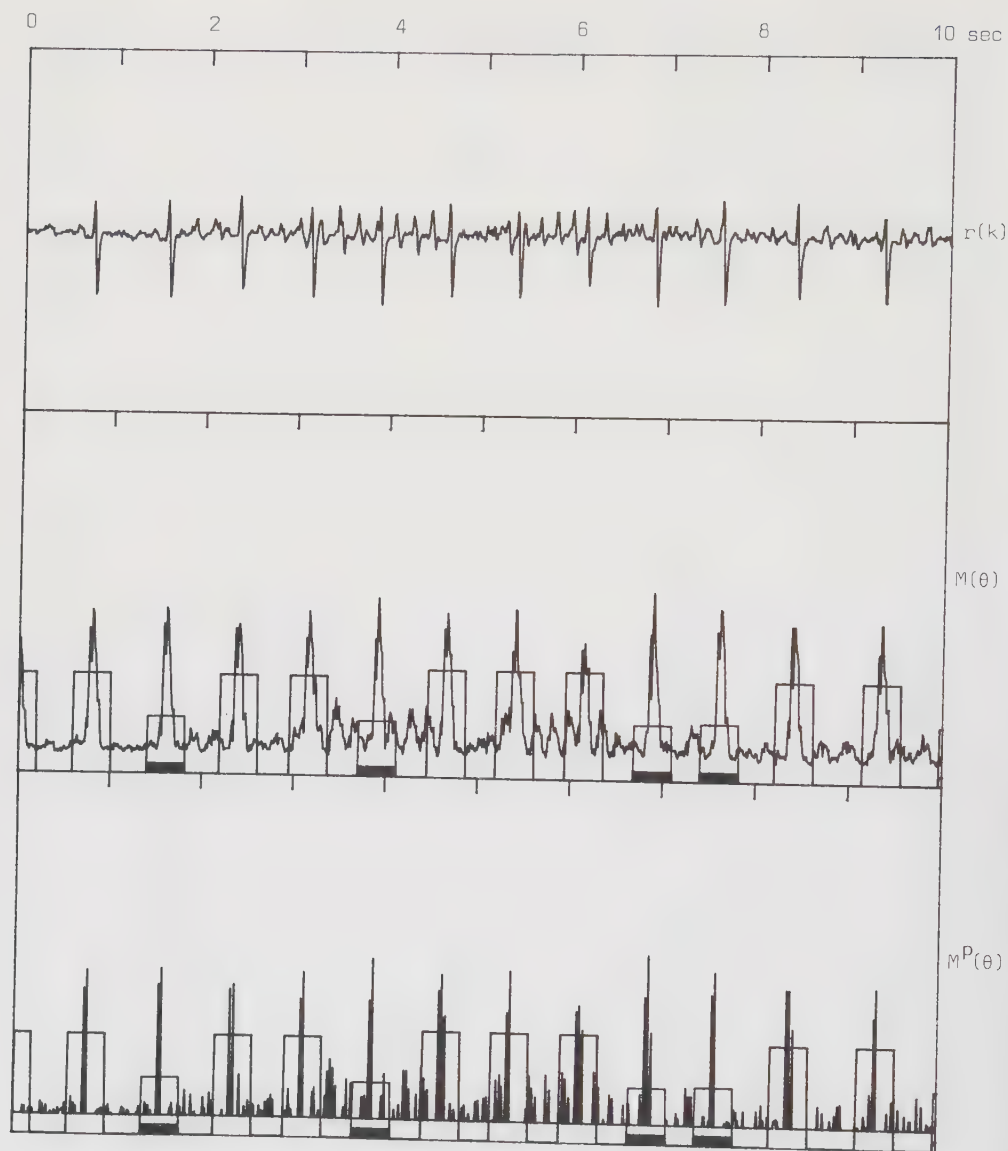


Fig. 17. QRS-like artefacts intermingled with normal QRS complexes.

Adaptive QRS detection: performance

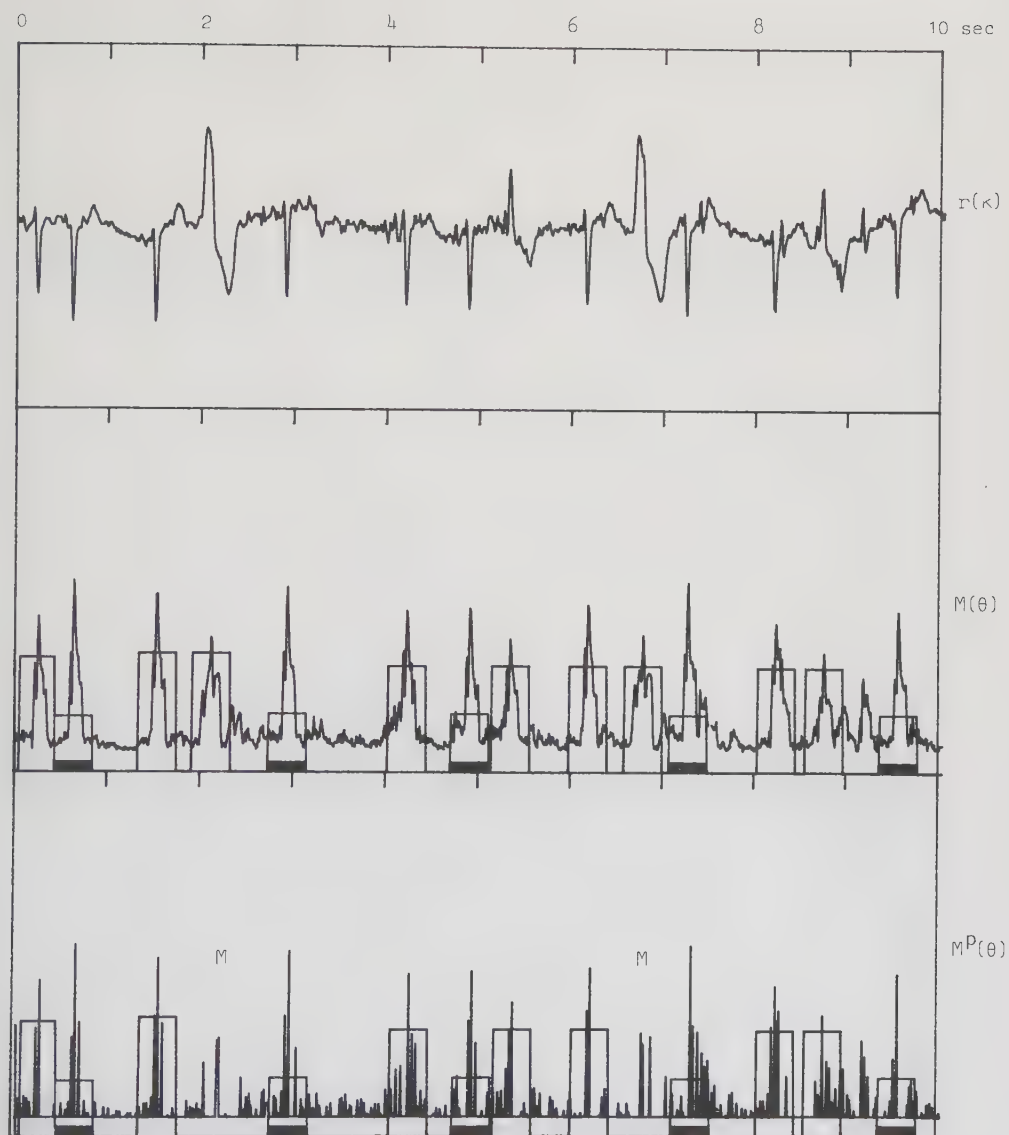


Fig. 18. Wide PVCs which are missed by PMAP.

Adaptive QRS detection: performance

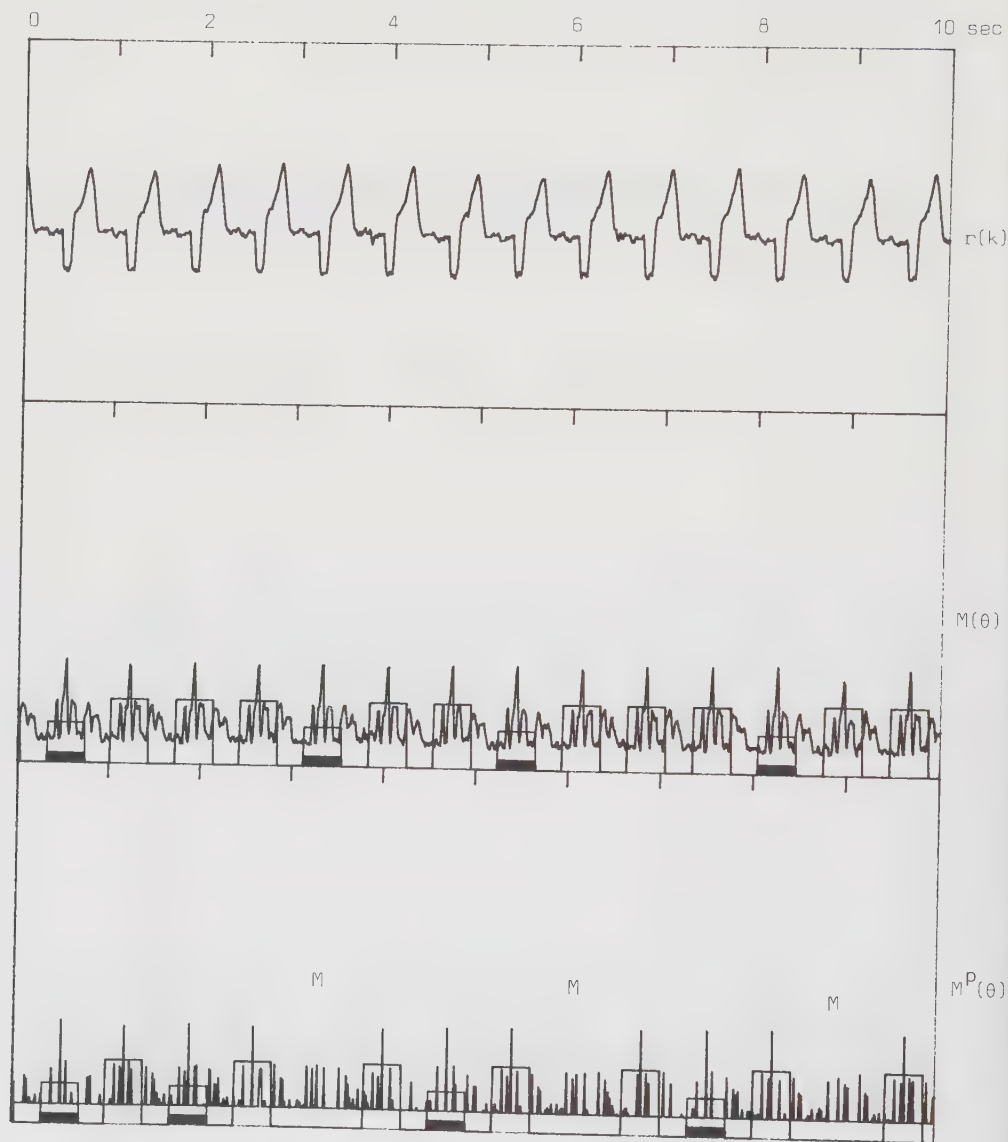


Fig. 19. Wide QRS complexes which are missed by PMAP.

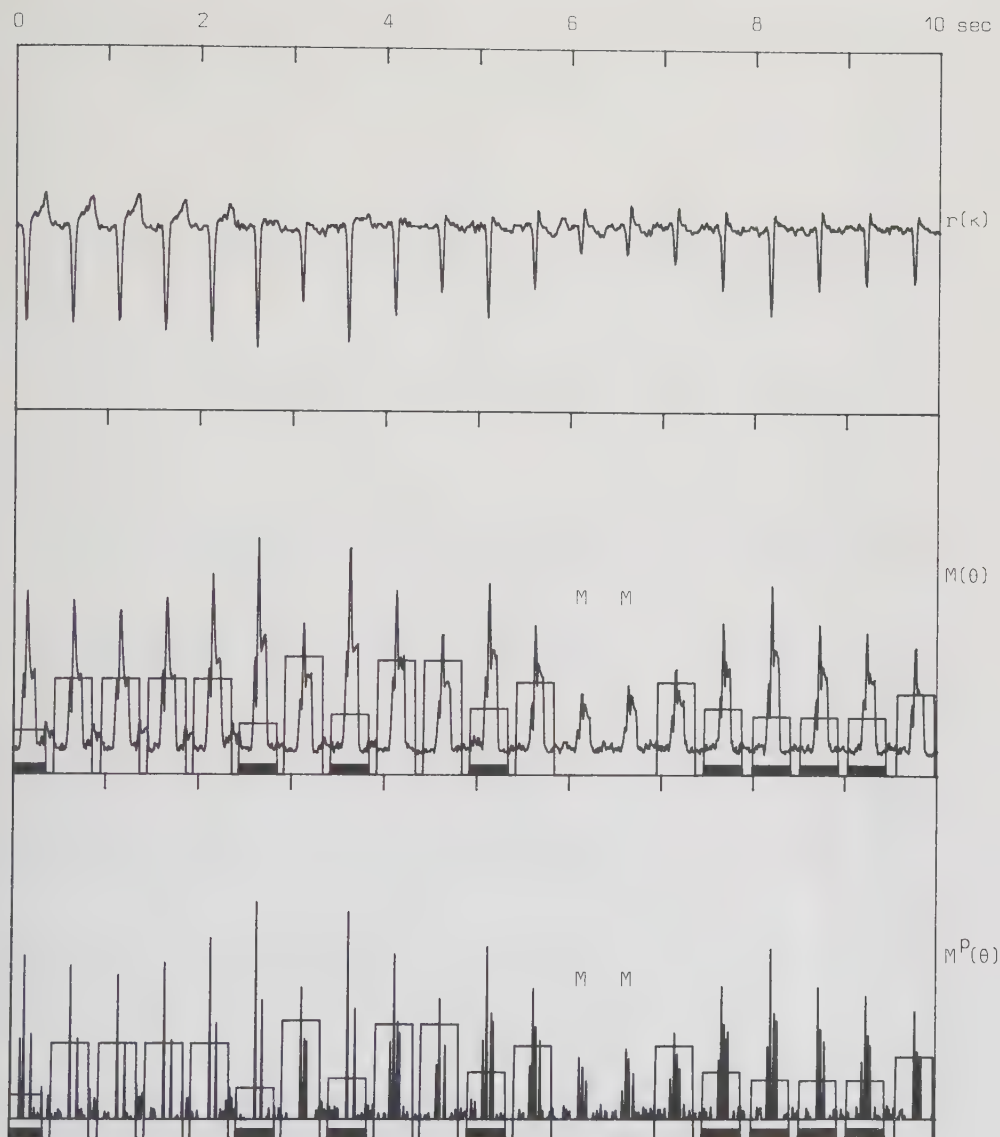


Fig. 20. Sinus tachycardia with small variation in the R-R intervals and rapidly changing QRS amplitude.

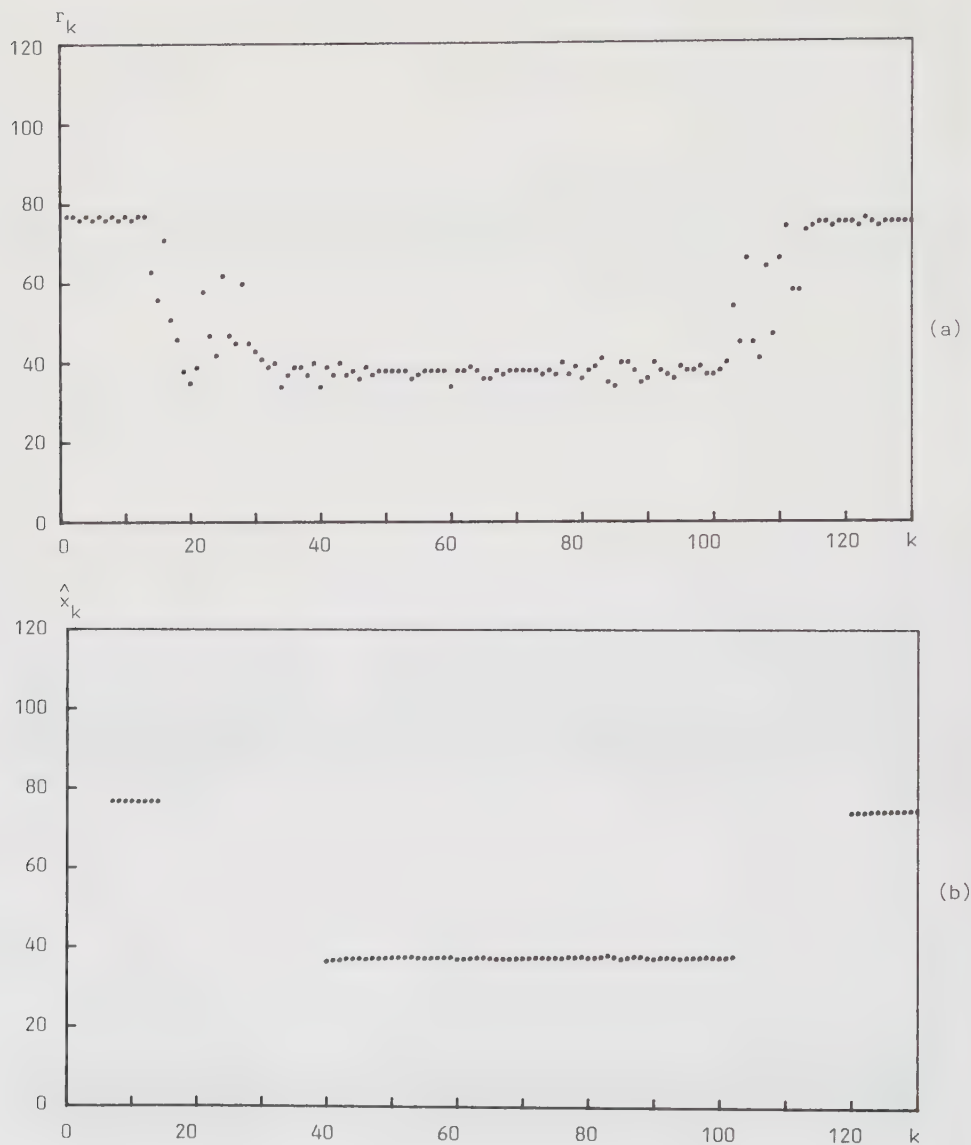


Fig. 21. Normal sinus rhythm interrupted by a short episode of tachycardia (a). "Empty" regions in (b) indicates that the rhythm is considered irregular.

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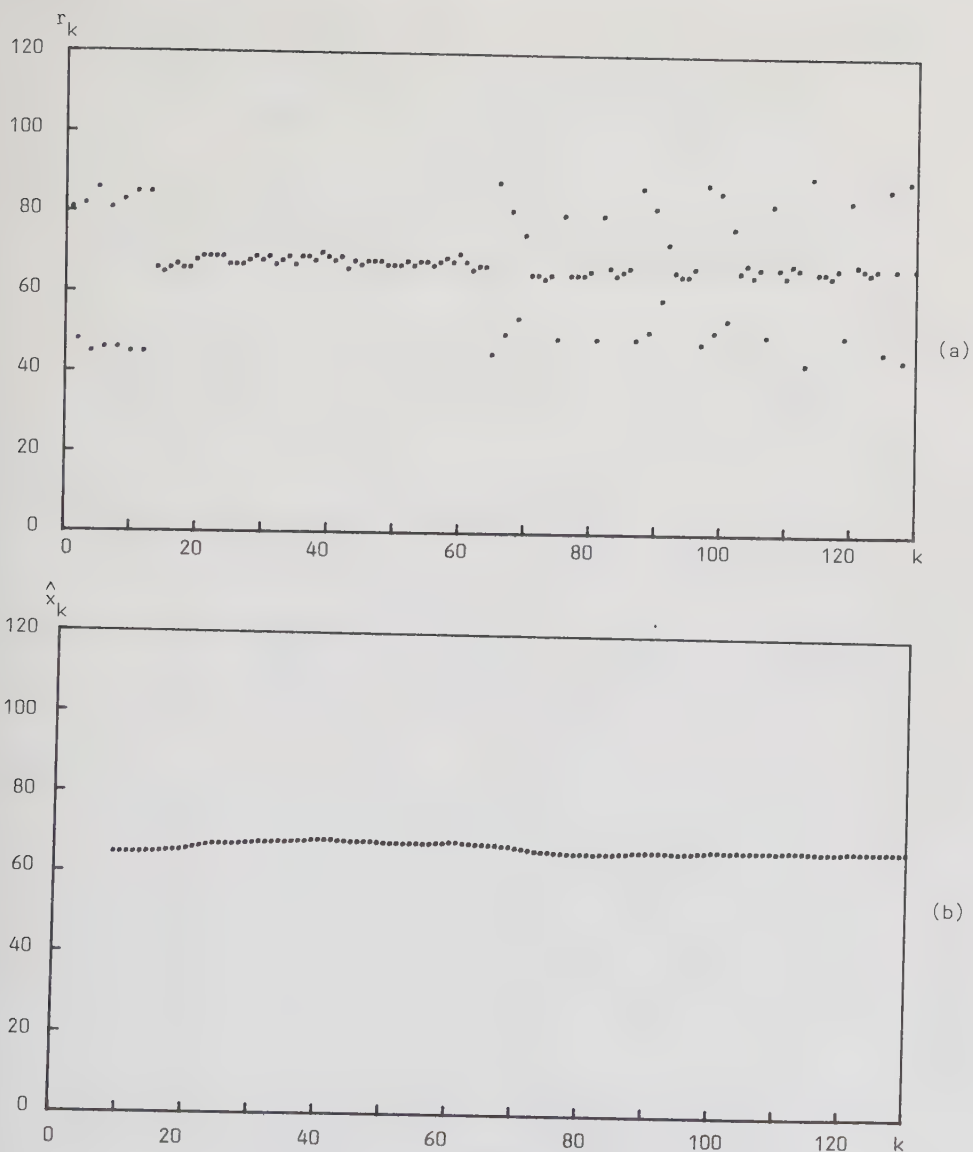


Fig. 22. Premature ventricular beats with compensatory pause intermingled in a normal sinus rhythm (a). An underlying regular rhythm is detected (b).

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A MODEL-BASED APPROACH
TO
QRS DELINEATION

PART III

ABSTRACT

QRS delineation is considered as a problem of finding changes in the spectral behaviour of an ECG. Based on statistical models for the "low-frequency" and "high-frequency" segments of the ECG, the points for QRS onset and end are obtained from a maximum-likelihood procedure. This procedure is based on the properties of the prediction errors obtained from two different Kalman filters. The accuracy of the delineation scheme is studied on a QRS complex material in terms of agreement with manually obtained delineations. The material was independently delineated by three different readers. The sensitivity to noise is studied by adding white noise to the QRS complexes. The performance is compared to those of envelope-based delineation and template waveform delineation.

I. INTRODUCTION

The recognition of the onset and end of QRS complexes is necessary in most types of computer-based ECG analysis. The performance of algorithms for such delineations may influence the performance of the analysis system as a whole.

In the analysis of resting ECGs, QRS delineation is one of the cornerstones of the subsequent measurement programs which measure QRS duration, Q wave duration etc. Inaccurate QRS delineation leads to erroneous measurements and may thus lead to erroneous diagnoses.

In exercise ECG analysis interest is focused on the presence or absence of changes in the ST-segment. ST changes are measured at certain pre-defined distances from the end of the QRS complex; accurate determination of QRS end is thus essential. The reference level for ST measurements is usually estimated from a segment just before QRS onset, the accurate determination of which is also important.

QRS delineation is, moreover, a prerequisite in certain schemes for classification of different QRS morphologies, found e.g. in arrhythmia analysis for ambulatory or coronary care monitoring. In these schemes, the features which characterize the QRS complex are computed with respect to the delineated interval. A summary of features which are based on the recognition of QRS onset and end is found in [1]. Delineation schemes which produce inaccurate estimates of QRS onset and end severely limit the reliability of the subsequent classification.

Estimation of QRS onset and end in multiple-lead recordings is usually based on a delineation function which is obtained by combining the different leads. Several types of delineation functions have been described in the literature [2]-[6], of which the most commonly employed is the spatial velocity. Since the delineation function usually is a positive-valued function, the recognition of QRS onset and end can be done by a simple threshold technique. Another technique is that of template matching, in which a template waveform is cross-correlated to the delineation function [4], [7]-[9]. The templates for QRS onset/end describe the average behaviour of the delineation function at these points, and are usually derived from a learning set in which the QRS complex-

es beforehand have been manually delineated. In ambulatory ECG analysis, QRS delineation schemes have instead often been based on searches for an appropriate combination of slopes [10]. The delineation in such cases has usually been performed in single-lead recordings. Although most systems for ambulatory ECG analysis employ QRS delineation, e.g. for feature extraction, very few schemes have been described in detail.

Despite the fact that QRS delineation has been employed in various systems for twenty years, no method has yet emerged which has gained wide acceptance. A comparison between different programs for analysis of resting ECGs indicates that large differences exist in the measurement of QRS duration [10]. Such discrepancies may be due to differences in design philosophy, but can also be due to systematic differences among cardiologists in defining QRS onset and end. A joint project is in progress in the European Communities (EC) with the purpose of establishing standardized definitions of various measurements in the ECG, e.g. QRS onset and end [11]. The database consists of 250 multiple-lead resting ECG recordings, delineated by five different cardiologists. This database serves as a unique tool in the development of accurate delineation schemes or as a test set for comparing the performance of different schemes.

A characteristic feature of the ECG is the change in spectral properties that occurs at the onset and end of the QRS complex. The abruptness of such changes is strongly dependent on the angle from which the heart is viewed. The abruptness of QRS onset, furthermore, depends on whether the electric impulse, which gives rise to the QRS complex, originates from a supraventricular or ventricular focus. In the latter case, the change is less marked since the electric potential is spread more slowly through the heart. The delineation scheme described in this part is based on the fact that any heart-beat can be decomposed into two segments with different spectral properties: low-frequency segments which either contain the P-wave or the T-wave, and a high-frequency segment, the QRS complex. This segmentation is exemplified in Fig. 1. These signal properties are both modeled by linear systems which are driven by white noise. The most likely point in time for a change is obtained by using maximum-likelihood (ML) estimation. The QRS onset is then defined as the point for when a change occurs from the low- to the high-frequency segment. A change in the reverse direction

defines the end of the QRS complex. In Section IV, the accuracy of the delineation scheme is studied in terms of agreement with the delineations obtained from three human readers. The performance is compared to both that of envelope-based delineation [12] and template waveform delineation [4].



Fig. 1. Segmentation of a heart-beat cycle in terms of low- and high-frequency segments.

II. MODELING OF THE ECG SIGNAL

In this section, the ECG model used for estimation of QRS onset will be described; QRS end is treated in a similar fashion, cf. Sec. III.2. The onset Θ is that point after which a change takes place in the observed discrete-time signal,

$$z(k) = \begin{cases} z_1(k) & k = 1, \dots, \Theta \\ z_2(k) & k = \Theta+1, \dots, N \end{cases} \quad (2.1)$$

where $z_1(k)$ and $z_2(k)$ are processes with different spectral and statistical properties. Both these signals are modeled by a linear system, which is defined by

$$x_i(k+1) = F_i(k) x_i(k) + G_i w_i(k) \quad (2.2)$$

$$\begin{aligned} z_i(k) &= H_i^T x_i(k) + v(k) \\ &= y_i(k) + v(k) \quad i = 1, 2. \end{aligned} \quad (2.3)$$

The state vector $x_i(k)$, G_i and H_i^T are $n_i \times 1$ vectors and $F_i(k)$ is a time-varying $n_i \times n_i$ matrix. The linear system is driven by the noise $w_i(k)$. The observations $z_i(k)$ consist of the desired signal characteristic $y_i(k)$ additively corrupted by the noise $v(k)$. Both system and observation noise are assumed to be white, zero-mean, Gaussian and mutually uncorrelated to each other. Furthermore, the covariances of the noise are

$$E[w_i(k) w_i(l)] = Q_i(k) \delta_{kl}, \quad i = 1, 2 \quad (2.4)$$

$$E[v(k) v(l)] = R \delta_{kl} \quad (2.5)$$

where δ_{kl} is the Kronecker delta. It should be noted that the variance R of $v(k)$ does not depend on whether a change has occurred or not. The initial conditions $x_1(0)$ and $x_2(\Theta)$ are $N(m_1(0), P_1(0))$ and $N(m_2(\Theta), P_2(\Theta))$, where $P_i(k)$ is an $n_i \times n_i$ matrix.

Model 1 represents the segment which precedes the QRS complex. The model accounts for the transition from a P-wave to a slowly varying baseline, by letting both spectral and statistical properties be simple functions of time. A first order system was considered for which

$$F_1(k) = \begin{cases} F_{11} & k = 1, \dots, \theta_p \\ F_{12} & k = \theta_p + 1, \dots \end{cases} \quad (2.6)$$

where $0 < F_{11} < F_{12} < 1$. The scalar G_1 is time-invariant and H_1 is equal to one. Systems of higher order were considered. However, no significant improvement in the agreement of automated and manual delineations was found. The variance of the input noise $w(k)$ is constant up to a time θ_p , after which it decreases linearly to zero

$$Q_1(k) = \begin{cases} q_1 & k = 1, \dots, \theta_p \\ q_1 \frac{k - N_D}{\theta_p - N_D} & k = \theta_p + 1, \dots, \theta_p + N_D \\ 0 & k = \theta_p + N_D + 1, \dots \end{cases} \quad (2.7)$$

The parameter N_D defines the width of the transition zone. For the moment, it is assumed that the time θ_p , which determines the beginning of the transition zone, is located on the isoelectric segment between the P-wave and QRS onset. In practice, the point θ_p is estimated from the observed signal; the estimation is described in Sec. III. The parameter values ($F_{11}, F_{12}, G_1, q_1, N_D$) were considered as design parameters, see Sec. IV. The choice of initial conditions is not particularly critical: therefore, $m_1(0)=0$ and $P_1(0)$ was set to a large value due to the possibility that the beginning of the observation interval may occur e.g. in the P-wave.

Since the shape of a QRS complex may differ considerably during the course of a recording, it would be desirable to identify the model parameters before each particular delineation. However, since such an identification presupposes that θ is known, an "over-all" QRS model has instead been identified which is used for delineation. The identification of F_2, G_2 and H_2 was done by first estimating the parameters $\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m$ and the orders n and m in the autoregressive moving average (ARMA) difference equation

$$y_2(k) = - \sum_{j=1}^n \alpha_j y_2(k-j) + \sum_{l=0}^m \beta_l w_2(k-l) \quad (2.8)$$

and then transforming the ARMA model to a state-space representation. The parameter β_0 is equal to 1 in (2.8), and the order $m < n$. Several articles

QRS delineation

have been published in which the problem of identifying the parameters of an ARMA model is treated. For an AR process ($m=0$) of known order, it is well-known that the mean square estimates $\hat{\alpha}_i$ are obtained as the solution of the Yule-Walker equation. Having determined parameter estimates for a "high" order AR-model, a procedure is described in [13], in which the parameter estimates of the ARMA-model are obtained from those of the AR-model by solving two simple algebraic equations. Criteria for determining the orders n and m are also given.

In order to estimate the parameters α_i and β_i in the "over-all" QRS model, a material of manually delineated QRS complexes, $y_2^j(k)$, was used, cf. Sec. IV.2 (sampling rate $f_s=250\text{Hz}$). The correlation function was estimated by

$$r_y(l) = \frac{1}{M} \sum_{j=1}^M \frac{1}{L_j} \sum_{k=1}^{L_j-|l|} y_2^j(k) y_2^j(k+|l|) \quad (2.9)$$

where L_j is the width of j :th QRS complex and M the number of QRS complex. A fifth-order model was found to be adequate for which the power spectrum of the output process is shown in Fig. 2.

So far the models have been developed such that the behaviour of $z_1(k)$ does not influence that of $z_2(k)$. No information is thus included in model 2 about the baseline level from which the QRS complex departs. In order to introduce this, the observed signal after a change is instead obtained by

$$z_2(k) = H_2^T x_2(k) + b + v(k) \quad (2.10)$$

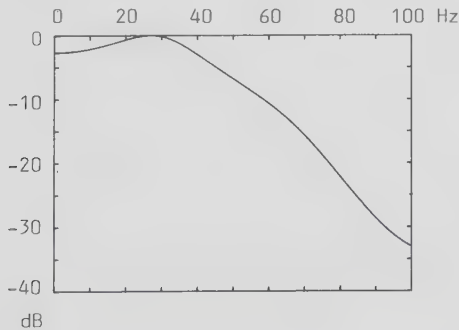


Fig. 2. Power spectrum for the output process from model 2.

where the baseline level b is taken as

$$b = x_1(\theta). \quad (2.11)$$

With the above modification, a suitable choice of initial condition for model 2 is that $x_2(\theta)$ is zero-mean and the covariance $P_2(\theta)$ is equal to the null matrix.

The assumption of the noise $v(k)$ being white is motivated by the influence of the electrical activity of the muscles in the ECG. In rough terms, this activity could be modeled as a process with constant spectral density up to a approximately 100Hz. If desired, however, the model could be extended to include a colored noise situation [14]; this will be at the expense of a more complex structure of the estimator.

Since the ECG already has been subject to QRS detection, the observation interval $[1, N]$ could be chosen such that (2.1) always is valid, i.e. θ is contained in the interval. Thus, the onset θ is constrained to the interval $1 < \theta < N$, ensuring that at least one sample is observed from each model. The end of the observation interval was taken as the time halfway between the two largest slopes around the peak of the QRS complex. The beginning of the interval was chosen at a fixed distance before the estimate of θ_p .

III. MAXIMUM LIKELIHOOD DELINEATION

III.1 Estimation of QRS onset

An ML-procedure is applied for estimating the point Θ since it is considered to be an unknown parameter. The estimation procedure is performed subsequent to having obtained estimates of the noise variance R and the point Θ_p ; the complete estimation procedure is summarized in Fig. 3. The ML-estimation of Θ is described first since the estimation procedures for R and Θ_p are similar in nature.

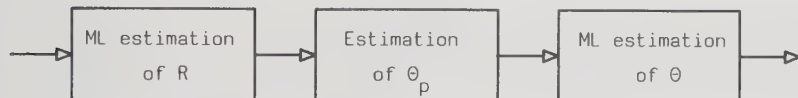


Fig. 3. Block diagram of estimation procedure for QRS onset.

The ML-estimate $\hat{\Theta}_{ML}$ is given by that Θ which maximizes the log-likelihood function

$$\Lambda(\Theta; Z(N)) = \ln p(z(1), \dots, z(N) | \Theta) \quad (3.1)$$

where $Z(N)$ is partitioned according to (2.1), and $p(\cdot | \Theta)$ denotes the joint probability density function of $Z(N)$ conditioned on Θ . Since both $z_1(k)$ and $z_2(k)$ are correlated Gaussian sequences, the maximization of $\Lambda(\Theta; Z(N))$ includes the inversion of several matrices. The required number of inversions quickly becomes unwieldy as N grows. To avoid this problem, each sequence is transformed into a zero-mean, white sequence assuming a change at Θ . Since such a transformation in our case is defined by a causal linear operation, no information will be lost. For the models described in Section II, the Kalman filter performs such a transformation. A brief summary of the equations defining the Kalman filter is here motivated. The filter yields, in the minimum-variance sense, the optimal estimate of the state-vector $x(k)$ based on the sequence $Z(N)$. It should be noted that this optimality is valid for a non-stationary model and a finite observation interval. The filter is defined by the following recursive equations [14].

$$\hat{x}(k|k-1) = F(k) \hat{x}(k-1|k-1) \quad (3.2)$$

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K(k) \tilde{z}(k|k-1) \quad (3.3)$$

$$\tilde{z}(k|k-1) = z(k) - H^T \hat{x}(k|k-1) - b \quad (3.4)$$

The filter is initialized with $\hat{x}(k_0|k_0) = m(k_0)$. The filtered estimate $\hat{x}(k|k)$ is obtained from the one-step prediction $\hat{x}(k|k-1)$ corrected with the prediction error $\tilde{z}(k|k-1)$ weighted with the gain $K(k)$. The filter gain can be computed in advance by the recursive equations

$$\Sigma(k|k-1) = F(k) \Sigma(k-1|k-1) F^T(k) + GQ(k)G^T \quad (3.5)$$

$$V(k) = H \Sigma(k|k-1)H^T + R \quad (3.6)$$

$$K(k) = \Sigma(k|k-1)H^T V^{-1}(k) \quad (3.7)$$

$$\Sigma(k|k) = \left(I - K(k)H \right) \Sigma(k|k-1) \quad (3.8)$$

which are initialized by $\Sigma(k_0|k_0) = P(k_0)$. The matrix $\Sigma(k|k-1)$ denotes the error covariance of the state estimate $\hat{x}(k|k-1)$. For observations originating from the model, the sequence $\tilde{z}(k|k-1)$ is zero-mean and white with variance $V(k)$.

By processing the sequence $Z(N)$ with the filters for model 1 and 2, the log-likelihood function (3.1) can, conditioned on the change at θ , be expressed as a product of random variables $\tilde{z}_i(k|k-1)$,

$$\Lambda(\theta; \tilde{Z}(N)) = \ln \sum_{k=1}^{\theta} p\left(\tilde{z}_1(k|k-1)|\theta\right) \prod_{k=\theta+1}^N p\left(\tilde{z}_2(k|k-1)|\theta\right). \quad (3.9)$$

By incorporating the Gaussian structure of the sequence $\tilde{Z}(N)$ in (3.9), one obtains

$$\Lambda(\Theta; \tilde{Z}(N)) = \sum_{k=1}^{\Theta} \left(\frac{\tilde{z}_1^2(k|k-1)}{V_1(k)} + \ln V_1(k) \right) + \sum_{k=\Theta+1}^N \left(\frac{\tilde{z}_2^2(k|k-1)}{V_2(k)} + \ln V_2(k) \right). \quad (3.10)$$

The ML estimate of Θ is then obtained by

$$\hat{\Theta}_{ML} = \arg \min_{\Theta} \Lambda(\Theta; \tilde{Z}(N)). \quad (3.11)$$

For convenience, an additive constant which is independent of Θ has been omitted in (3.10). It should be noted that at each time $(\Theta+1)$ a new filter is initialized in order to produce $\tilde{z}_2(\Theta+1|\Theta)$, $\tilde{z}_2(\Theta+2|\Theta+1), \dots$. The variance $V_2(k)$ is thus not defined for $k < \Theta+1$.

The noise variance R is estimated from the observations (model 1) in an interval $[\ell_0, \ell_1]$ located before the QRS complex. The resulting estimate is then used for both estimation of QRS onset and end. The ML estimate of R is obtained by minimizing the log-likelihood function

$$\Lambda(R; \tilde{Z}(N)) = \sum_{k=\ell_0}^{\ell_1} \left(\frac{\tilde{z}_1^2(k|k-1)}{V_1(k)} + \ln V_1(k) \right) \quad (3.12)$$

with respect to R . If a high accuracy of the estimate $\hat{R}_{ML}$ is required, a gradient search procedure generally converges faster to the prescribed neighbourhood of the minimum than does a direct search procedure. The gradient of $\Lambda(R; \tilde{Z}(N))$ is computed from a system of equations (see e.g. [15]), which substantially increases the total amount of computations. In our situation, it was found that the requirements on the accuracy of $\hat{R}_{ML}$ could be relaxed without influencing $\hat{\Theta}_{ML}$ in a significant way. Therefore, a direct search procedure provided the least time-consuming alternative. The search was performed under the constraint that $\hat{R}_{ML} \geq R_0$, in order to prevent the Kalman filter from overweighting the most recent observation. By omitting this constraint, $\hat{R}_{ML}$ is close to zero if the P-wave is absent and the SNR is high. It should be noted that (3.2)-(3.8) are recomputed in the search procedure for each new value of R .

QRS delineation

The estimate of θ_p affects the gain $K(k)$ in filter 1 such that the observations $z(\hat{\theta}_p+1)$, $z(\hat{\theta}_p+2)$... are weighted less and less, thus resulting in an increase of the prediction errors $\tilde{z}_1^2(k|k-1)$ subsequent to QRS onset. The estimation of θ_p is based on the normalized short-term energy of the prediction errors for model 1,

$$\gamma(\theta_p) = \frac{\sum_{k=\theta_p-W}^{\theta_p+W} \tilde{z}_1^2(k|k-1)}{V_1(k)} \quad (3.13)$$

where $(2W+1)$ is the length of the data window. The sequence $\gamma(\theta_p)$ is for each θ_p computed by restarting the filter at each time (θ_p-W) . Note that the change in (2.6)-(2.7) occurs in the middle of each window $[\theta_p-W, \theta_p+W]$. The restarting of the filter is motivated by the desire to avoid large P-waves (not accurately modeled by model 1) to obscure subsequent predictions. An estimate of θ_p is then obtained by searching backwards in $\gamma(\theta_p)$ from $\theta_p=N$ until a constant threshold, η , is crossed.

At higher noise levels, the estimate of QRS onset may sometimes be located prior to a spurious waveform, which thus is interpreted as a Q wave. In order to reduce such problems, the strength of the QRS complex (i.e. the variance Q_2 of the system noise) was related to the estimate of R by

$$Q_2 = \lambda \cdot \hat{R}_{ML}. \quad (3.14)$$

A larger Q wave is thus required at an increased noise level if it should be included in the delineated interval.

The constant b in (3.4) is equal to zero in model 1. An estimate of b in (2.11) is obtained by using the last one-step prediction in model 1,

$$\hat{b} = \hat{x}_1(\theta|\theta-1) \quad (3.15)$$

III.2 Estimation of QRS end

By reversing the ECG signal, the estimation procedure for QRS onset is applied equally for the location of QRS end. Model 1 accounts in this case instead for the presence of the T-wave. Although the isoelectric segment prior to the QRS complex has no counterpart in the reversed signal, still

a rather distinct change is commonly found subsequent to the S-wave. Since the estimation of QRS onset and end takes place independently of each other, it is possible to use different sets of parameter values in each of the estimation procedures.

This section is concluded with three different examples which illustrate maximum likelihood delineation, see Fig. 4.

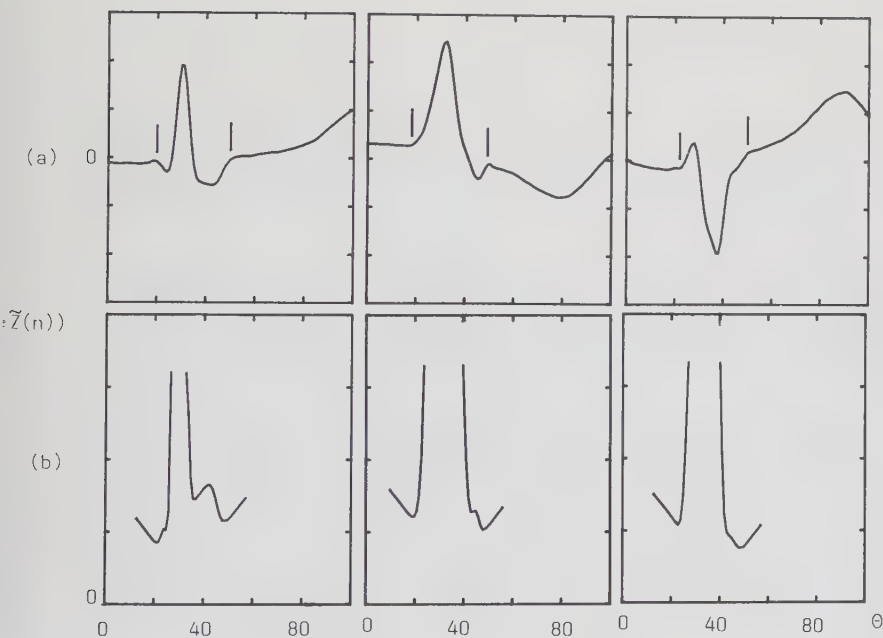


Fig. 4. Three examples of maximum likelihood delineation.

(a) original ECG shown with estimates of QRS onset and end,
 (b) the log-likelihood functions $\Lambda(\theta; \tilde{Z}(N))$ for onset and end, respectively. The estimates indicated in (a) are given by the minima in these functions.

IV. ACCURACY OF AUTOMATED QRS DELINEATION

In order to judge the accuracy of a QRS delineation scheme, the agreement between manual and automated delineation must be investigated. Since the definition of QRS onset and end sometimes is unclear, e.g. due to high-frequency noise, consistent delineation among human readers cannot be expected. By letting a number of skilled readers delineate a QRS complex, the uncertainty associated with the particular delineation could be estimated.

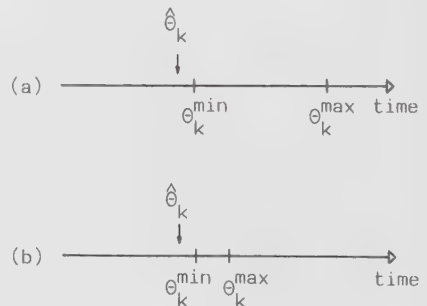
IV.1 Performance measure

We will now describe a performance measure which relates the automated delineation to the manual ones. A sequence of points in time $\{\theta_k^1, \dots, \theta_k^h\}$ is obtained by h different human readers having determined QRS onset, or end, for QRS complex k . The largest value of this sequence is denoted with $\theta_k^{\max}$ and the smallest value with $\theta_k^{\min}$. The following properties of the performance measure were desired:

1. Since each time instance θ_k^i should be considered to be equally true, any estimate $\hat{\theta}_k$ which is such that $\theta_k^{\min} \leq \hat{\theta}_k \leq \theta_k^{\max}$ should be judged correct.
2. The seriousness of an erroneous estimate $\hat{\theta}_k$ should be related to the length of the interval $[\theta_k^{\min}, \theta_k^{\max}]$, see Fig. 5.
3. The error in $\hat{\theta}_k$ should be judged equal, when $\hat{\theta}_k$ occurs at the same distance from $\theta_k^{\max}$ as from $\theta_k^{\min}$.

Fig. 5.

- (a) If the length of the interval $[\theta_k^{\min}, \theta_k^{\max}]$ is large as compared to $(\hat{\theta}_k - \theta_k^{\min})$, the inaccuracy of $\hat{\theta}_k$ should be considered less serious because the uncertainty among human readers is large.
- (b) Alternatively, if the length of the interval is small as compared to $(\hat{\theta}_k - \theta_k^{\min})$, the inaccuracy of $\hat{\theta}_k$ should be considered more serious.



A performance measure which complies with the above properties is defined by

$$\epsilon_k = \frac{\epsilon'_k}{1 + \theta_k^{\max} - \theta_k^{\min}} \quad (4.1)$$

where ϵ'_k is an error variable which is given by

$$\epsilon'_k = \begin{cases} \hat{\theta}_k - \theta_k^{\min} & \hat{\theta}_k < \theta_k^{\min} \\ 0 & \theta_k^{\min} \leq \hat{\theta}_k \leq \theta_k^{\max} \\ \hat{\theta}_k - \theta_k^{\max} & \hat{\theta}_k > \theta_k^{\max} \end{cases} \quad (4.2)$$

Note that a too early estimate of QRS onset or end results in a negative ϵ_k and vice versa.

IV.2 QRS complex material

A material of QRS complexes was obtained from 126 non-selected patients in a coronary care unit [12]. The ECGs were recorded via the normal monitoring lead - usually with the electrodes placed over the sternum - and amplified by conventional bedside monitors (bandwidth 0.1-40Hz). One representative QRS complex for each morphological category was selected and digitized from a two-minute tape recording for each patient using a sampling rate of 500Hz. In this study, the sampling rate was decimated to 250Hz. A total of 200 complexes were collected, 69 of which were ventricular ectopic beats. Each digitized QRS complex was independently delineated by three experienced readers. QRS onset and end were reported by aligning a marker on a high-resolution graphics terminal (Tektronix 4012) to the desired positions.

In order to describe the consistency of the manual delineations, the cumulative number of QRS complexes for which the interval $[\theta_k^{\min}, \theta_k^{\max}]$ is less or equal than a certain number of samples was determined, see Fig. 6. The diagram indicates, somewhat surprisingly, that it was slightly more difficult to establish onset consistently. Naturally, the dispersion of θ_k^i is larger for slow changes in the initial and terminal part of the QRS complex. In some cases, an initial deflection was either interpreted as a Q wave or as part of the baseline variation.

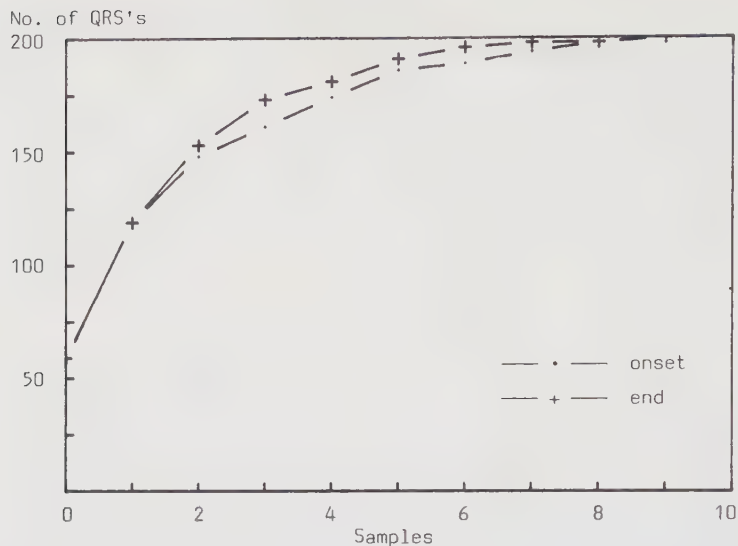


Fig. 6. The cumulative number of QRS complexes for which the interval $[\theta_k^{\min}, \theta_k^{\max}]$ is less or equal than a certain number of samples.

IV.3 Results - "noise-free" ECGs

The performance of maximum likelihood delineation (MLD) was compared to both that of envelope-based delineation (ED) and template waveform delineation (TWD). These two algorithms are briefly described in Appendices A and B, respectively. The parameter values of $(F_{11}, F_{12}, G_1, q_1, N_D, \lambda, W, \eta)$ in MLD were chosen so as to produce a good over-all agreement with QRS onset in the above material (in this case, QRS onset was independently defined by a fourth reader). The following values were used: $F_{11}=0.95, F_{12}=0.99, G_1=0.05, q_1=150, N_D=3, \lambda=1000, W=7, \eta=25$. Note that G_1 and q_1 are coupled via (3.5). The above parameter values were then, for simplicity, also used when estimating QRS end. The parameter values used in ED and TWD are given in the Appendices.

Histograms have been obtained which show the distribution of the performance variable ϵ_k for MLD, ED and TWD, see Fig. 7. The average of ϵ_k and $|\epsilon_k|$ are given for each diagram. The number of correct estimates, n_c , is not shown in the histogram, but is given separately. Note that the area of the histogram is inversely related to n_c ; if n_c is equal to 200 the area is zero.

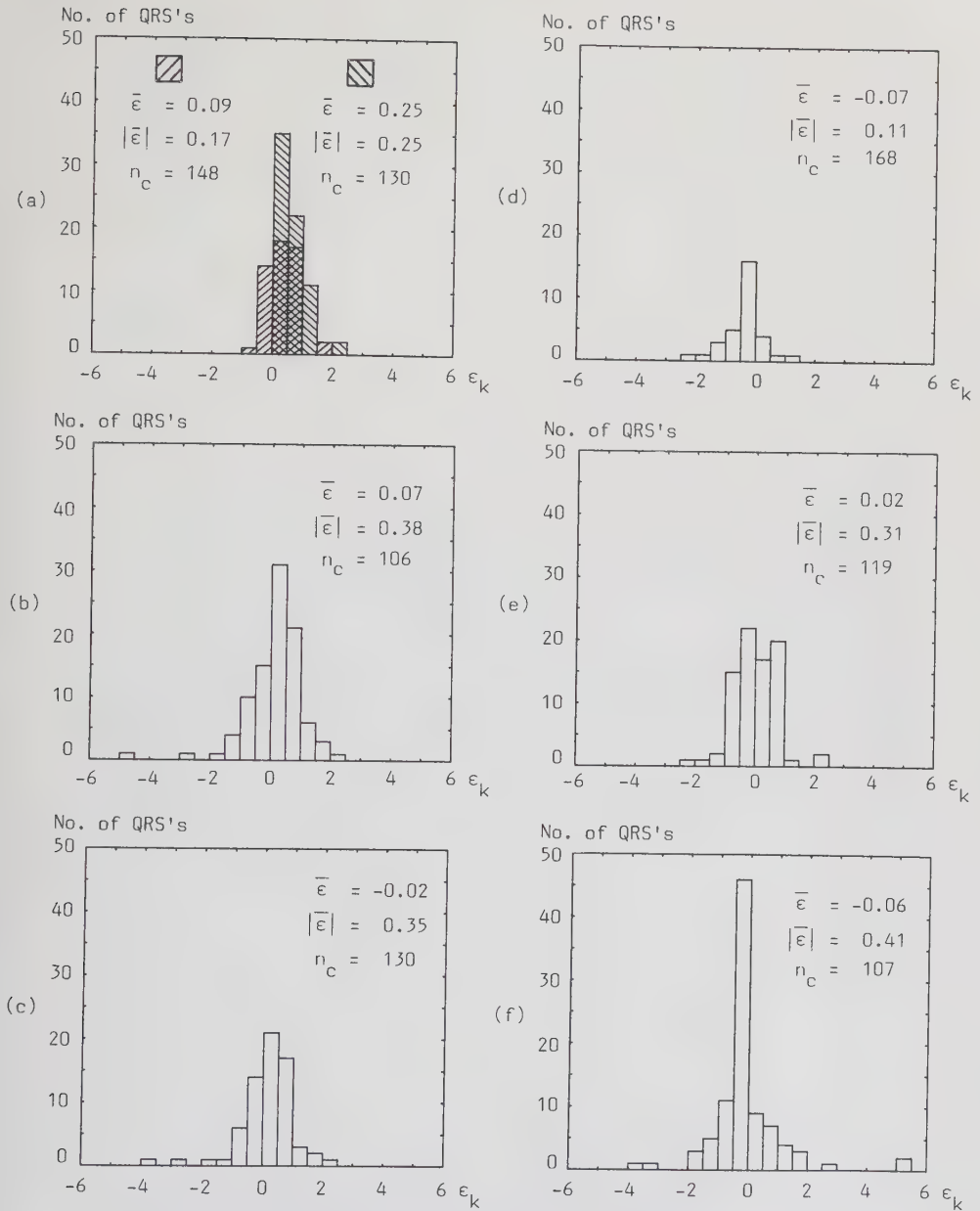


Fig. 7. The distribution of ϵ_k for QRS onset by using (a) MLD (▨) and MLD modified with "one-sample decrease" (▩), see text, (b) ED, (c) TWD and for QRS end by using (d) MLD, (e) ED and (f) TWD.

The estimates of QRS end were for MLD and ED in general more accurate than those of QRS onset, while the reverse relationship was found for TWD. The results indicate that MLD yielded the smallest average error $|\bar{\epsilon}|$ both for onset and end. In particular, estimation of QRS end by MLD resulted in an average error which was substantially smaller than by using either ED or TWD. The distribution of ϵ_k for QRS onset with MLD differs considerably in shape as compared to those of ED and TWD. While estimates from MLD typically occur at QRS onset or afterwards, i.e. $\epsilon_k \geq 0$ for all k , estimates from ED and TWD are such that ϵ_k is more evenly distributed around zero. By introducing a fixed one-sample decrease of the estimates from MLD, the distribution of ϵ_k becomes more centered around zero, see Fig. 7(a).

The estimation of onset in cases of a slowly changing initial phase of the QRS complex is exemplified in Fig. 8(a)-(b). The estimates from MLD are in both examples more accurate than those from ED and TWD, the latter estimates occurring well inside the QRS complex. However, if the amplitude of the largest wave is decreased, the estimate of QRS onset from ED and TWD will be affected, see Fig. 9. In order to demonstrate this property, the samples in an interval centered around the peak with largest magnitude was decreased by different weights. The magnitude of the initial deflection was kept unchanged. Note that while the estimate of onset from MLD is independent of the peak magnitude, the estimates from ED and TWD move towards QRS

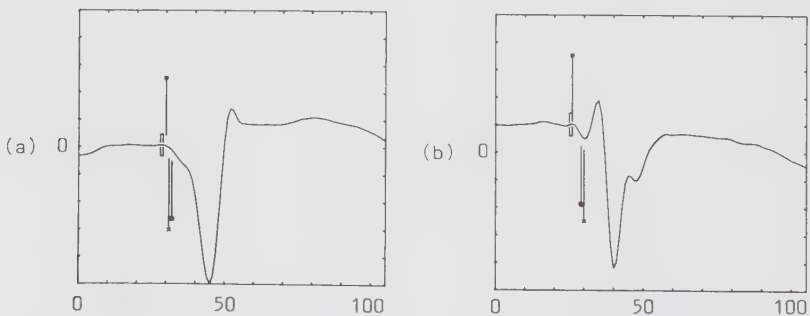


Fig. 8(a)-(b). Estimation of QRS onset in cases with slow initial deflections. The range of manual delineations is marked with a box. $\square$ - MLD, $\circ$ - ED, $\times$ - TWD.

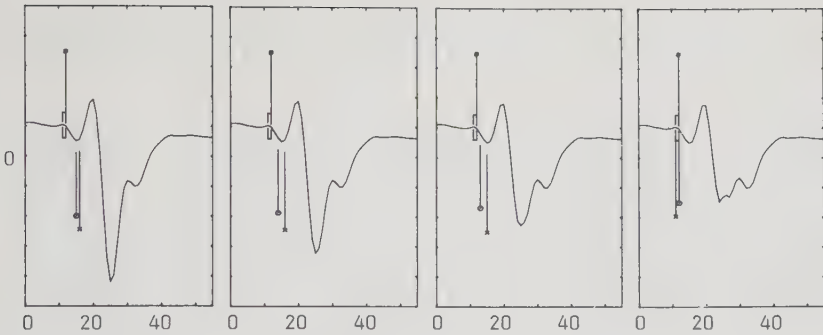


Fig. 9. The effect of decreasing the S-wave amplitude while keeping the Q-wave unchanged. The range of manual delineations is marked with a box. $\square$ - MLD, $\circ$ - ED, $\times$ - TWD.

onset for decreasing magnitudes. For the low-amplitude beats contained in the material, ED and TWD produced estimates occurring several samples before the manual delineations. This property is due to the normalization of the signal: in ED it is accomplished by incorporating the peak amplitude of the envelope in the threshold (cf. (A.6)), and in TWD by normalizing the delineation function $d_{SV}(n)$, see p. 161. In MLD the signal "strength" Q_2 is only related to the noise level via the multiplicative factor λ , see (3.14). The effect on $\bar{\epsilon}$ and $|\bar{\epsilon}|$ for different values of λ is shown in Fig. 10. While the choice of λ does not affect $|\bar{\epsilon}|$ in a significant way, the bias becomes rather large for large λ . By choosing a large λ , however, the estimator becomes more immune to noise.

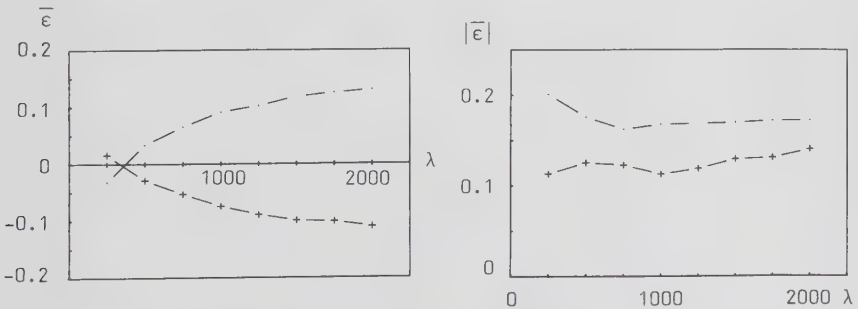


Fig. 10. The performance measures $\bar{\epsilon}$ and $|\bar{\epsilon}|$ as functions of the constant λ in MLD.

Very small Q-waves, for which it may be difficult to establish a distinct onset, presented problems for all three delineation schemes. In such cases, the onset was usually estimated close to the nadir of the wave.

The length of the iso-electric segment between the end of the P-wave and QRS onset may influence the estimation of onset. The effect of different P-wave locations relative to the QRS complex is demonstrated in Fig. 11. The estimate from MLD remained unaffected by the P-wave as long as the segment length was larger than W, which in this case was equal to 7 samples. Note that it is essential to restart the Kalman filter in (3.13) at each point in time. The estimate from ED was unaffected by the presence of a P-wave in all of the four cases. For segments of length shorter than 20 samples, TWD identified the onset of the P-wave as QRS onset. The behaviour of TWD is essentially due to the lowpass filtering of the signal, which thus decreases the resolution between the P-wave and the QRS. This problem could be reduced by increasing the cut-off frequency of the filter, thus making the delineation scheme more sensitive to noise, or better by employing adaptive filtering [9].

A majority of problems associated with the estimation of QRS end arise in situations when the transition from the QRS complex to the ST-segment is less marked. Examples of QRS complexes with such transitions are shown in Fig. 12. The terminal part of the QRS complex in Fig. 12(a) changes so slow-

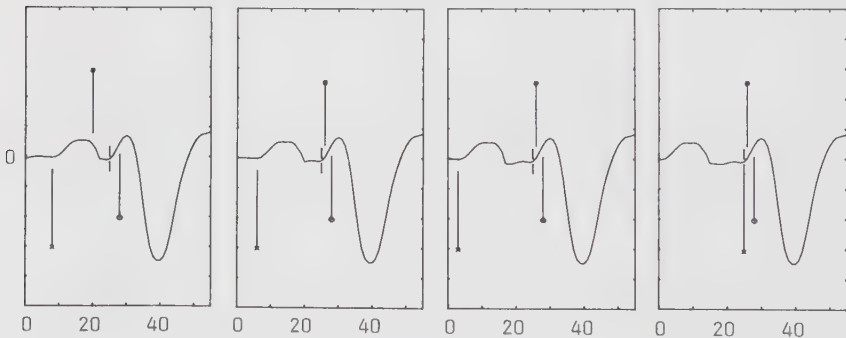


Fig. 11. The effect of different lengths of the isoelectric segment between the P-wave and the QRS complex. The range of manual delineations is marked with a box. $\square$ - MLD, $\circ$ - ED, $\times$ - TWD.

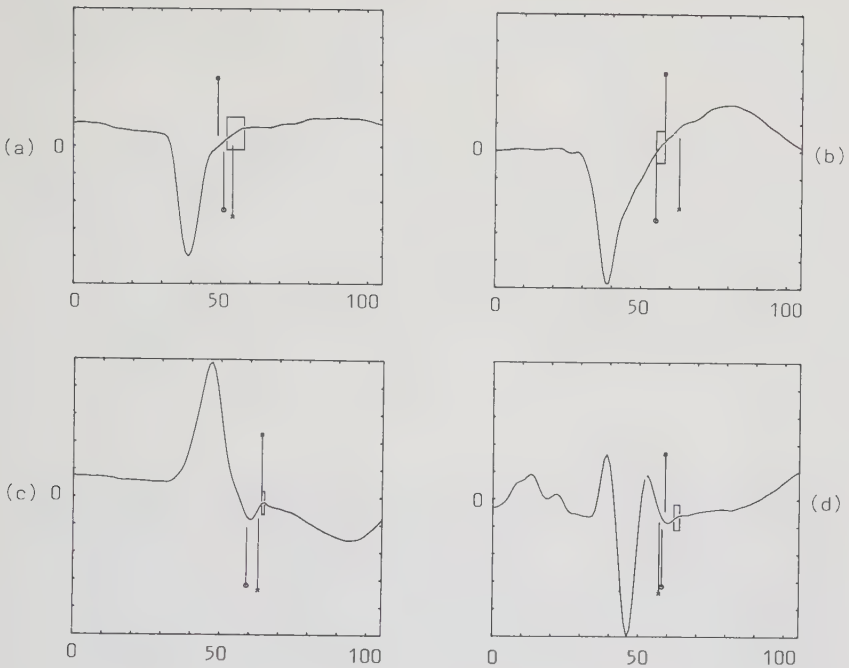


Fig. 12(a)-(d). Examples of estimation of QRS end. The range of manual delineations is marked with a box. $\square$ - MLD, $\circ$ - ED, $\times$ - TWD.

ly such that it is in MLD interpreted as belonging to model 1 (the estimate of θ_p is located within this part). The estimation of QRS end in Figs. 12(a)-(b) is difficult because its position depends not only on the curvature of the ST-segment but also on the relation to the baseline level. Thus in comparison with Fig. 12(a), QRS end in Fig. 12(b) is located on a steeper part of the slope since the baseline level has been crossed. No information about the baseline level is included in MLD for improving the accuracy of estimating QRS end in smooth segments. Such information could, however, be introduced by assuming that θ obeys some non-uniform distribution, the location of which is related to the crossing of the baseline level. The use of a maximum-a-posteriori criterion is then suitable for estimation of θ .

The correlation between the template and the "spatial velocity" $d_{SV}(n)$ is rather poor for the complex in Fig. 12(b) when estimating QRS end ($\varphi(\hat{\theta}) = 0.8$); a behaviour due to the fact that $d_{SV}(n)$ remains fairly large after

QRS end. This property contributed to the fact that $|\bar{\epsilon}|$ for TWD was larger for QRS end. When estimating onset instead, a stable baseline commonly exists prior to the QRS complex which thus results in a better fit to the template.

Another case for which it is difficult to establish QRS end is when the terminal part of the QRS complex is constituted by a slow wave, see Figs. 12(c)-(d). The estimates from ED and TWD occur in Fig. 12(c) inside the QRS complex, while that of MLD is contained in the interval of manual delineations. In Fig. 12(d), none of the delineation schemes produced an accurate estimate of end.

IV.4 Results - noisy ECGs

To gain knowledge about noise immunity of the QRS delineation schemes, the performance was studied by adding white Gaussian noise to the QRS complexes in the material. Although the noise in an ECG hardly can be considered neither Gaussian nor uncorrelated, the simulation procedure is still useful for studying the relative merits of MLD, ED and TWD in noisy recordings. Since all three schemes to various degrees account for baseline wandering, immunity to such noise was not investigated.

The performance in noisy ECGs was studied in terms of the difference

$$\Delta\epsilon_k = \epsilon_{k,\text{SNR}} - \epsilon_{k,\infty} \quad (4.3)$$

where $\epsilon_{k,\text{SNR}}$ is measured in the presence of noise and $\epsilon_{k,\infty}$ from the noise-free ECG. The standard deviation of the difference, $\sigma_{\Delta\epsilon_k}$, was for each QRS complex and SNR estimated from 25 different noise realizations. It should be noted that if $\epsilon_{k,\infty}$ is equal to zero, a standard deviation equal to zero does not necessarily imply that the estimate $\hat{\epsilon}_{k,\text{SNR}}$ is identical to $\hat{\epsilon}_{k,\infty}$ for all realizations, but that $\hat{\epsilon}_{k,\text{SNR}}$ is contained in the interval $[\epsilon_k^{\min}, \epsilon_k^{\max}]$.

The SNR was defined as a ratio between the variance $r_y(0)$, as estimated in (2.9) from the material, and the noise variance,

$$\text{SNR} = 10 \log \frac{r_y(0)}{R} \quad [\text{dB}] \quad (4.4)$$

An example of different SNRs is shown in Fig. 13.

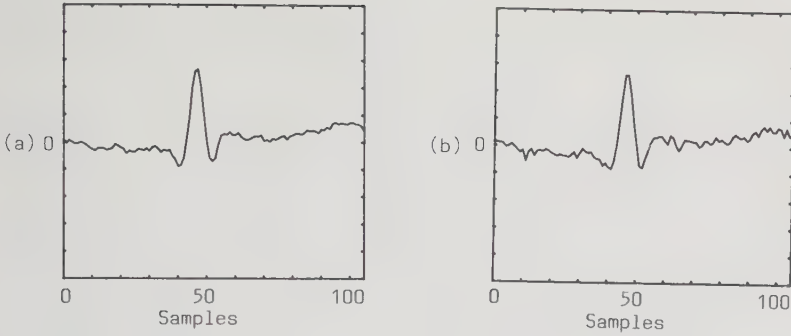


Fig. 13. Examples of different signal-to-noise ratios.

(a) 35 dB, (b) 30 dB.

The resulting standard deviation $\sigma_{\Delta\epsilon_k}$ for each of the 200 QRS complexes is shown in Fig. 14 for the different delineation schemes (for ease of interpretation, $\sigma_{\Delta\epsilon_k}$ is rearranged in increasing order for each scheme). The diagrams indicate that ED, both for onset and end, produced those estimates for which $\Delta\epsilon_k$ was smallest. While the performance of MLD differed little from that of TWD for onset, a smaller dispersion of $\Delta\epsilon_k$ was obtained for QRS end.

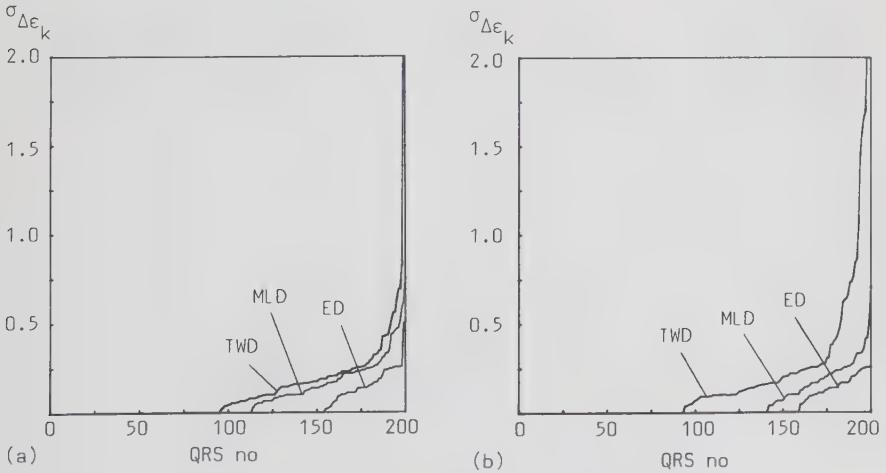


Fig. 14. The standard deviation $\sigma_{\Delta\epsilon_k}$ of the difference between ϵ_k as measured in noise-free and in noisy ECGs for each of the QRS complexes in the material. (a) onset, 35 dB, (b) end, 35 dB, (c) onset, 30 dB and (d) end, 30 dB.

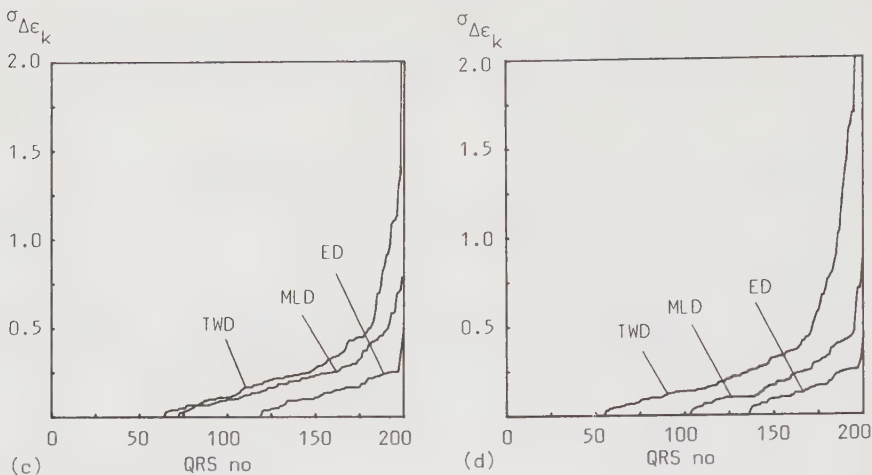


Fig. 14. Continued.

For MLD and TWD a rather large dispersion of $\Delta \epsilon_k$ occurred for QRS complexes with a small initial/terminal deflection, in such cases causing the estimate to alternate between onset/end of the deflection and its nadir. For some of those complexes, however, the desire to minimize the dispersion is not self-evident, since the noise may obliterate a small deflection. This fact is illustrated by Fig. 15, where that QRS complex is shown for which the largest dispersion at onset occurred for MLD. While the dispersion of TWD was similar to that of MLD, it was equal to zero for ED. Although the initial deflection has virtually disappeared in Fig. 15(c), the estimate for ED remains un-

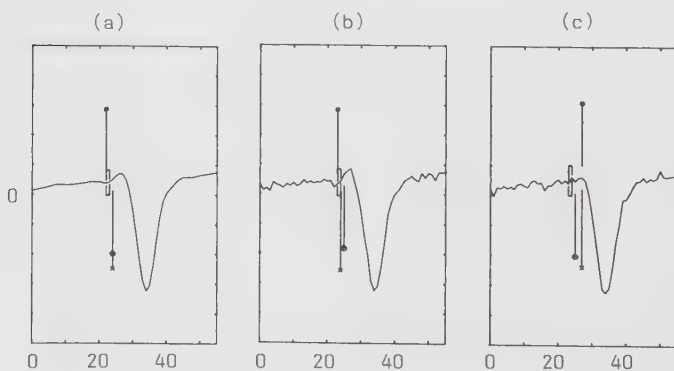


Fig. 15(a)-(c). Examples of performance for different noise realizations added to a noise-free ECG; SNR = 30 dB. The range of manual delineations' is marked with a box. $\square$ - MLD, $\circ$ - ED, $\times$ - TWD.

changed, see Fig. 15(b) and (c). The noise-immune property of ED is due to the lowpass filtering combined with the modification procedure for the points of threshold crossings, see Appendix A. Such a property is achieved at the expense of a large systematic error for certain QRS morphologies, see Fig. 16. By using e.g. a lowpass filter with higher cut-off frequency in (A.2), i.e. for a smaller M_2 , systematic errors of the type seen in Fig. 16 will decrease. The use of such a filter will, however, also increase the sensitivity of ED to noise.

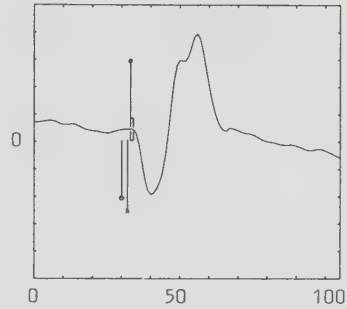


Fig. 16. Example of QRS morphology in which a large systematic error occurs in ED. The range of manual delineations is marked with a box. $\square$ - MLD, $\circ$ - ED, $\times$ - TWD.

V DISCUSSION

QRS delineation is in this study viewed as a problem of detecting when a change between two statistical models occurs. Similar approaches have been considered for the purpose of segmenting e.g. electroencephalograms (EEGs) and seismic signals into segments with different spectral properties, e.g. [16], [17], [18]. Since the observation interval in the latter applications usually is considerably longer than in the QRS delineation case, the model parameters may be estimated from the observed signal. While most estimation procedures are based on the properties of the prediction errors, some recent methods instead rely on the properties of the estimated model parameters, [17], [19]. In the MLD scheme, predefined values of the model parameters were used for delineating all the QRS complexes, except for the estimated parameters R and Θ_p . The choice of parameter values does not seem to be particularly critical, since those values which were used for estimation of QRS onset also were appropriate for estimation of end; in fact, the average error $|\bar{e}|$ was smaller for QRS end than for onset, see Fig. 7(a) and (d).

The variability of QRS delineation in noisy signals was studied by adding white noise to the QRS complexes in the material. It is also of interest to study the effect on QRS delineation when noise of non-stationary character is present, such as electrode motion artefacts. Since the noise variance R is estimated in MLD under the assumption of white noise, it is necessary to modify the noise estimation procedure before MLD is applied in such situations.

The performance of a delineation scheme is in this study described by the variable ϵ_k . With this measure, the performance is made dependent on the consistency of manual delineations. Since the material was delineated by three independent readers, large differences resulted in some cases, see Fig. 6. By letting the readers instead discuss and adjust their individual definitions of QRS onset and end, a higher degree of consistency is most likely obtained. It is thus, in the latter case more difficult to achieve a small $|\bar{e}|$. It was in this study believed that the material with independently obtained manual delineations could serve as the initial step in studying the performance of different schemes.

QRS delineation

The QRS complex material was not divided into a learning set and a test set. Thus, further validation of the schemes seems motivated before a definite statement could be made on performance. However, the results still indicate the relative merits of the delineation schemes, since all three schemes took advantage of the material in the parameter tuning phase (see Sec. IV.3 and Appendix A and B).

APPENDIX A: ENVELOPE-BASED QRS DELINEATION

The various processing steps involved in envelope-based QRS delineation [12] are described by the block diagram in Fig. 17. Each step is briefly summarized below.

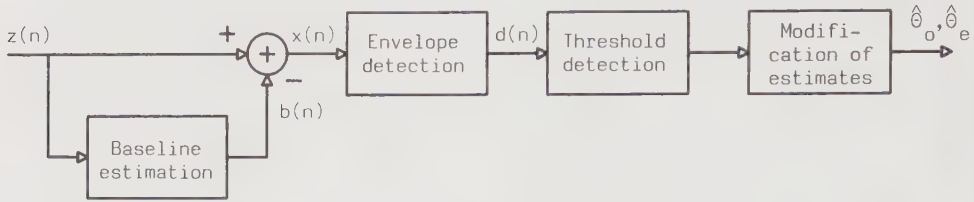


Fig. 17. Block diagram describing the steps of envelope-based QRS delineation ($\hat{\theta}_o$ and $\hat{\theta}_e$ denote estimates of QRS onset and end, respectively).

1. Envelope detection. The envelope $x_e(n)$ of a real-valued sequence $x(n)$ is defined by

$$x_e(n) = \left(x^2(n) + \hat{x}^2(n) \right)^{1/2} \quad (\text{A.1})$$

where $\hat{x}(n)$ is the discrete-time Hilbert transform of $x(n)$. Before the envelope can be computed, an approximation of the ideal Hilbert transformer, must be designed. It was found that an approximate Hilbert transform filter with short duration was suitable for QRS delineation. Furthermore, the envelope in (A.1) was approximated by the sum of the absolute values of $x(n)$ and $\hat{x}(n)$.

In noise-free ECGs, the envelope of a QRS waveform usually exhibits a monotonic behaviour in the regions of threshold crossings. However, noise corrupting the ECG results in a certain ripple in $x_e(n)$. In order to avoid ambiguities in the threshold detection, $x_e(n)$ is lowpass filtered by using a filter with symmetric, triangular impulse response $h_{LP}(k)$,

$$d_E(n) = \sum_{i=-M_2}^{M_2} x_e(n-i) h_{LP}(i) \quad (\text{A.2})$$

where $2M_2+1$ is the duration of $h_{LP}(k)$. This filter could be implemented without any multiplications by means of two cascaded averaging filters.

2. Baseline estimation. The baseline is obtained by weighted averaging, using coefficients which depend on the pattern of the signal. The baseline $b(n)$ is computed from the ECG, $z(n)$, in a moving window of $2L_2+1$ points

$$b(n) = \frac{\sum_{i=-L_2}^{L_2} w(n-i) z(n-i)}{\sum_{i=-L_2}^{L_2} w(n-i)} \quad (A.3)$$

where the weights $w(n)$ are chosen so as to favour the segments between the QRS complexes. To achieve this, a preliminary delineation function $d'(n)$ is defined as the envelope of the first-difference of $z(n)$. The weights $w(n)$ are then defined by

$$w(n) = 1 / \left(d'(n) + c d'_p \right) \quad (A.4)$$

where c is a positive constant and d'_p is the peak amplitude of $d'(n)$.

3. Threshold detection. QRS onset and end are estimated from the points where $d(n)$ crosses a threshold. Since the presence of noise causes an increase in the "background" level of $d(n)$, a dynamic threshold is used to make the delineation scheme immune to noise while still permitting accurate delineation of small Q and S waves in a noise-free situation. The threshold is defined by

$$\beta(n) = \gamma \cdot d_p + d_0(n) \quad (A.5)$$

where $0 < \gamma < 1$ and d_p is the peak amplitude of $d(n)$. The background level $d_0(n)$ is obtained by weighted averaging of $d(n)$ according to (A.3)-(A.4). In this case the functions $z(n)$ and $d'(n)$ are both replaced by $d(n)$, and d'_p is replaced by d_p .

4. Modifications of estimates. At higher noise levels the "tails" of the QRS complex will be hidden in the background level of $d(n)$. Consequently, the distance between the threshold crossings represents a truncated estimate of QRS duration. On the other hand, the lowpass filtering in (A.2) tends to broaden the QRS complex. To compensate for these effects, the onset and end are estimated from the points of threshold crossing assuming a symmetric

trapezoidal pulse as a model for the envelope of the QRS. In particular, the lowpass filtered trapezoidal pulse is first scaled with respect to width and amplitude so that it coincides with the observed delineation function in the points of threshold crossings and has the same peak amplitude. The width of the trapezoidal pulse is chosen as a fraction (b_T) of the distance between the threshold crossings. The onset and end of the corresponding trapezoidal pulse are then taken as estimates of QRS onset and end.

Envelope-based QRS delineation was studied with those parameter values used in [12], which were determined from part of the material, ($M_2=8$, $L_2=37$, $c=0.05$, $\gamma=0.08$, $b_T=0.35$). The delineation procedure is illustrated for three different QRS complexes in Fig. 18.

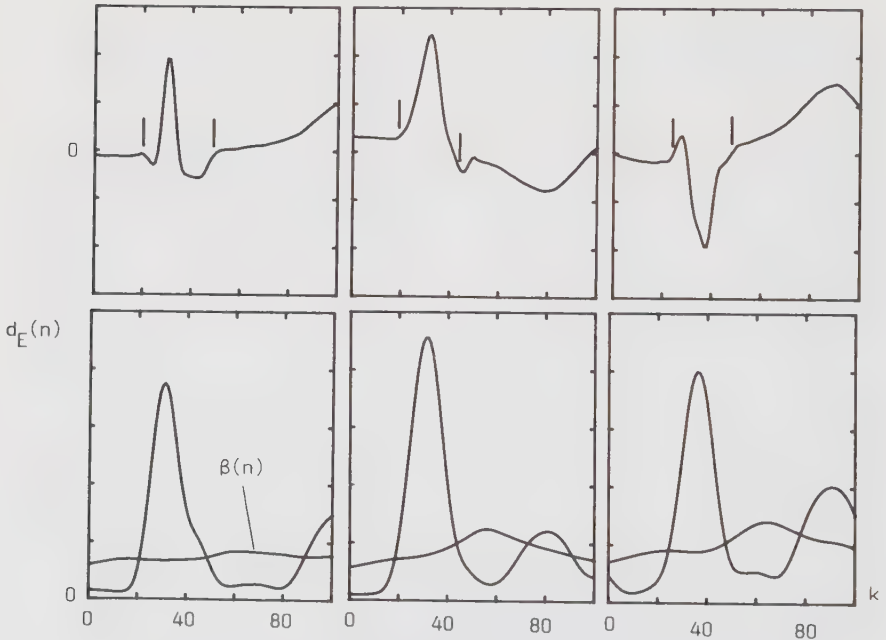


Fig. 18. Three examples of envelope-based delineation. (a) original ECG shown with the estimates of QRS onset and end, (b) the delineation function $d_E(n)$ and the threshold $\beta(n)$. Preliminary estimates of onset and end are determined from the threshold crossings. The final estimates are obtained from a modification procedure which is based on these crossings, see text for details.

APPENDIX B: TEMPLATE WAVEFORM DELINEATION

The delineation functions given in Table I on p. 19 exhibit a similar characteristic for most types of QRS waveforms in the vicinity of onset and end. By learning the "average" characteristic from a manually delineated set of waveforms, correlation techniques could be used for estimating QRS onset and end. A two-dimensional cross-correlation procedure was presented by van Bommel et al. [4], [9], in which the templates take into consideration both time and amplitude properties of the QRS complex. Although their method was applied to a 3-lead configuration, it is readily adapted to single-lead recordings. Below follows a short summary of the template waveform delineation scheme.

The delineation function $d_{SV}(n)$, which is defined in Table I on p. 19, is computed from the lowpass filtered ECG (the filter having spectral zeroes at 50, 75, ..., 200 Hz; sampling rate 250 Hz). The function $d_{SV}(n)$ is normalized for each QRS complex such that $d'_{SV}(n) = d_{SV}(n)/d_{SV,max}$.

Before cross-correlation can take place, the template for QRS onset is determined from a set of manually delineated QRS complexes (in our case, this was done by using the material described in Sec. IV.2 and the delineations as defined by a fourth reader). The template for QRS end is determined in a similar manner. For each waveform, a binary function (+1 or -1) is first produced by clipping of $d'_{SV}(n)$ with a threshold at the level α_i ($0 < \alpha_i < 1$). The template corresponding to this level is then obtained by averaging the set of aligned binary functions,

$$h(k,i) = \frac{1}{200} \sum_{j=1}^{200} \text{sign}[d'_{SV,j}(k+\theta_j) - \alpha_i] \quad (B.1)$$

where θ_j is the manually defined onset for QRS complex number j . The template $h(k,i)$ is computed for equidistant levels, $\alpha_i = \alpha_0 + i\Delta\alpha$, with the indices k and i constrained to the intervals $[k_0, k_1]$ and $[0, k_\alpha]$, respectively.

The estimate of QRS onset is that point in time, $\hat{\theta}$, for which the normalized correlation sum

$$\varphi(n) = \frac{\sum_{k=k_0}^{k_1} \sum_{i=0}^{k_\alpha} \text{sign}[d'_{SV}(k+n) - \alpha_i] h(k,i)}{\sum_{k=k_0}^{k_1} \sum_{i=0}^{k_\alpha} |h(k,i)|} \quad (\text{B.2})$$

is maximum. The number of different α_i 's is equal to $k_\alpha = \lfloor 1 + (\alpha_1 - \alpha_0) / \Delta\alpha \rfloor$. In certain situations, e.g. noisy ECGs, the resulting correlation factor $\varphi(\hat{\theta})$ may become very low, and thus indicating that the estimate is unreliable. To cope with this problem, the estimate was only accepted if $\varphi(\hat{\theta}) \geq \varphi_0$. The function $\varphi(n)$ was, otherwise, recomputed for a new set of levels α_1 , α_0 and α_1 being shifted upwards. This procedure is repeated for different α_0 and α_1 (each time both incremented with $\Delta\alpha_r$) until an estimate is found for which $\varphi(\hat{\theta}) \geq \varphi_0$. If no correlation factor exceeds φ_0 when α_0 reaches $\alpha_{\max}$, $\hat{\theta}$ is that estimate which corresponds to the largest $\varphi(\hat{\theta})$.

The delineation scheme was implemented with the parameter values given in [9], i.e. ($k_0 = -8$, $k_1 = 8$, $\alpha_0 = 0.026$ (onset), $\alpha_0 = 0.036$ (end), $\alpha_1 = 0.106$, $\Delta\alpha = 0.002$, $\Delta\alpha_r = 0.01$, $\alpha_{\max} = 0.106$, $\varphi_0 = 0.8$). Other choices of $[k_0, k_1]$ and $[\alpha_0, \alpha_1]$ were considered. As compared to the above choice, however, no significant improvement was found in terms of agreement to manual delineations. The correlation function $\varphi(n)$ is shown for three different QRS complexes in Fig. 19.

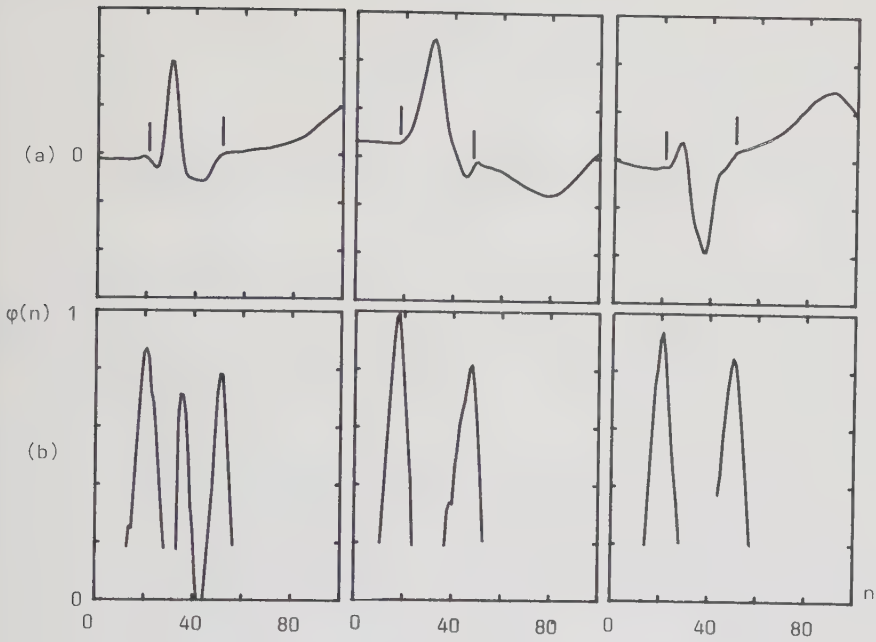


Fig. 19. Three examples of template waveform delineation. (a) original ECG shown with estimates of QRS onset and end, (b) the correlation functions $\phi(n)$ shown for onset and end, respectively. The estimates indicated in (a) are given by the maxima in these functions.

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